Assignment 1.1

The Fundamental Counting Principle and Permutations

Total: 45 marks

You need to show all your steps as you work out your solutions, and make sure your answers are clear (include units, where appropriate). Check the Introduction for the Marking Guidelines that will be used for all assignments and examinations.

1. Use the Fundamental Counting Principle to answer the following question. Using the digits 1, 2, 3, 5, 6, 8, 0, with no repetition allowed, how many four-digit numbers can be constructed if

   a) there are no other restrictions other than no repeated digits? Is the order of the digits important. Explain. *(3 marks)*

6 6 5 4 = 6×6×5×4 = 720 four digit numbers can be constructed with no repitition

4 spaces for each number, first space has 6 options since the number cannot start with a 0. Second spot includes the number 0, therfore it is a 6 as well. order is not important since they are multiplied and provide the same result.

   b) the number is even? *(3 marks)*

5 5 4 4 = 5×5×4×4 = 400 options for even numbers

5 options after eliminating the even # and counting out 0

counting 0

four options for even numbers. (2, 6, 8, 0)

4 options remaining

continued

Assignment 1.1: The Fundamental Counting Principle and Permutations (continued)

c) each digit of the number is even? *(2 marks)*

2 remaining options
1 remaining option

4 3 2 1 = 4 × 3 × 2 × 1 = 24

4 options (0, 2, 6, 8)

3 remaining options

continued

Assignment 1.1: The Fundamental Counting Principle and Permutations (continued)

2. Use the Fundamental Counting Principle to answer the following question. In how many ways can four women and four men be seated in a row on a large bench if

   a) there are no restrictions? *(1 mark)*

$8 \quad 7 \quad 6 \quad 5 \quad 4 \quad 3 \quad 2 \quad 1$

$= 40,320$

   b) the men and women must alternate seats? *(2 marks)*

$\underset{w}{4} \quad \underset{m}{4} \quad \underset{w}{3} \quad \underset{m}{3} \quad \underset{w}{2} \quad \underset{m}{2} \quad \underset{w}{1} \quad \underset{m}{1} = 576$

   c) one man and one woman are dating and want to sit together? *(2 marks)*

$\underset{m}{1} \quad \underset{w}{1} \quad 6 \quad 5 \quad 4 \quad 3 \quad 2 \quad 1 = 720$

↳ 6 people remaining and so forth.

   d) one man and one woman used to date and now they cannot sit beside each other? *(1 mark)*

$\underset{w}{1} \quad 6 \quad 5 \quad 4 \quad 3 \quad 2 \quad 1 \quad \underset{m}{1} = 720$

basically the same calculation. they would just sit elsewhere but 2 people would be accounted for so the other 6 people can sit wherever meaning there are 6 options.

continued

Assignment 1.1: The Fundamental Counting Principle and Permutations (continued)

3. Simplify without using a calculator. All work must be shown for marks.

a) $\frac{7!}{6!}$ *(1 mark)* $\frac{7 \times 6!}{6!} = 7$

b) $\frac{12!}{10!}$ *(1 mark)* $\frac{12 \times 11 \times 10!}{10!} = 12 \times 11 = 132$

c) $\frac{10!}{6!4!}$ *(1 mark)* $\frac{10 \times 9 \times 8 \times 7 \times 6!}{6!4!} = \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1} =$

d) $6!7$ *(1 mark)* $7 \times 6! = 7!$

e) $3!2!$ *(1 mark)* $3 \times 2 \times 1 \times 2 \times 1 = 12$
$(3 \times 2 \times 1)(2 \times 1)$

f) $\frac{5!8!}{4!7!}$ *(1 mark)* $\frac{(5 \times 4!)(8 \times 7!)}{4!7!} = 5 \times 8 = 40$

g) $\frac{(n+7)!}{(n+6)!}$ *(1 mark)* $\frac{(n+7)(n+6)!}{(n+6)!} = (n+7)$

2 h) $\frac{n!}{(n-2)!}$ *(2 marks)* $\frac{n(n-1)(n-2)!}{(n-2)!} = n(n-1)$

2 6

continued

Assignment 1.1: The Fundamental Counting Principle and Permutations (continued)

4. Solve for n.

a) $\frac{n!}{(n-1)!} = 10$ *(1 mark)*

$\frac{n(n-1)!}{(n-1)!} = n = 10$

b) $\frac{n!}{(n-2)!} = 42$ *(3 marks)*

$\frac{n(n-1)(n-2)!}{(n-2)!} = n(n-1) = 42$

$= n^2 - n = 42$

$= n^2 - n - 42 = 0$

5. Rewrite each in factorial notation and then find the value without using your calculator. *(4 × 2 marks each = 8 marks)*

a) ${}_4P_2$ $\frac{4!}{(4-2)!} = \frac{4!}{2!} = \frac{4 \times 2!}{2!} = 4$

formula $\frac{n!}{(n-r)!}$

b) ${}_7P_3$ $\frac{7!}{(7-3)!} = \frac{7!}{4!} = \frac{7 \times 6 \times 5 \times 4!}{4!} = 7 \times 6 \times 5 = 210$

c) ${}_6P_6$ $\frac{6!}{(6-6)!} = \frac{6!}{(0)!} = 6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$

d) ${}_{101}P_2$ $\frac{101!}{(101-2)!} = \frac{101!}{(99)!} = \frac{101 \times 100 \times 99!}{99!} = 10,000$

continued

Assignment 1.1: The Fundamental Counting Principle and Permutations (continued)

6. Explain your thinking.

   a) Manitoba Public Insurance requires that learners must pass a written test before receiving their Learner's License. The test consists of twenty multiple choice questions, of which sixteen must be answered correctly. Explain why this is not an application of permutations. *(1 mark)*

Permutation is the arrangment of a set of objects in which the order of the objects is important. In this case it does not matter which questions or in which order they are answered as long as a minimum of 16 are answered correctly.

   b) Describe, without calculating, how to find the number of ways three different jobs can be filled from amongst eight people. *(1 mark)*

8 7 6 = 336

$${}_nP_r = \frac{n!}{(n-r)!} = \frac{8!}{(8-3)!} = \frac{8!}{5!} = \frac{8\times7\times6\times5!}{5!} = 8\times7\times6 = 336$$

7. Find the number of different arrangements of the word FRIGHTEN if *(6 × 1 mark each = 6 marks)*

   a) there are no restrictions

$${}_8P_8 = \frac{8!}{0!} = 8! = 40{,}320$$

   b) the first letter must be G

$$\underset{G}{1} \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5040$$

   c) the first letter is not a G

40,320 − 5040 = 35280

$$7 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 35280$$

continued

Assignment 1.2

Permutations and Combinations

Total: 49 marks

You need to show all your steps as you work out your solutions, and make sure your answers are clear (include units, where appropriate). Check the Introduction for the Marking Guidelines that will be used for all assignments and examinations.

1. Determine whether each of the following situations represents a permutation or a combination. Explain. *(4 × 1 mark each = 4 marks)*

   a) 10 singers are competing at a local singing competition. How many different ways can the three top prizes be awarded?

   Since the order matters in this case for the top 3 prizes, it would be a permutation

   b) Three of your classmates are being selected to compete in a math competition. How many different ways can three people be chosen?

   Combination since the order of the three classmates does not matter in how they are placed in that group

   c) How many different ways can 25 students be assigned to desks in a classroom?

   Combination since the order of how the students are arranged to which desk does not matter

   d) A coach must choose five players to go to the all-star tournament. How many different groups of players can be chosen?

   Combination since the order in which the 5 players are selected does not matter

continued

Assignment 1.2: Permutations and Combinations (continued)

2. Evaluate using factorial notation. Show all work. *(4 × 2 marks each = 8 marks)*

a) $\binom{10}{3}$ $_{10}C_3$

$$\frac{10 \times 9 \times 8}{3 \times 2 \times 1} = \frac{720}{6} = 120$$

$$nCr = \frac{n!}{(n-r)!r!}$$

$$\frac{\frac{n!}{(n-r)!r!}}{\frac{n!}{(n-r)!}}$$

b) $\binom{5}{4}$ $_5C_4$

$$\frac{5 \times 4 \times 3 \times 2}{4 \times 3 \times 2 \times 1} = \frac{30}{6} = 5$$

c) $_8C_4$ $\frac{8!}{(8-4)!4!} = \frac{8!}{4!4!} = \frac{28 \times 7 \times 6 \times 5 \times 4!}{4 \times 3 \times 2 \times 1 \times 4!} = 70$

b) $_9C_6$ $\frac{9!}{(9-6)!6!} = \frac{9!}{3!6!} = \frac{9 \times 8 \times 7}{3 \times 2 \times 1} = \frac{504}{6} = 84$

3. a) Evaluate $_6P_4$. *(1 mark)*

$$\frac{6!}{(6-4)!} = \frac{6!}{2!} = \frac{6 \times 5 \times 4 \times 3 \times 2!}{2!} = 360$$

b) Evaluate $_6C_4$. *(1 mark)*

$$\frac{6!}{(6-4)!4!} = \frac{6!}{2!4!} = \frac{6 \times 5}{2} = \frac{30}{2} = 15$$

c) Which one of the above answers is larger? Explain using the definition of permutations and combinations. *(1 mark)*

Permutation since the order matters in that case So there will be more options. With the combination, the order does not matter so less available options are the result.

continued

d) the first letter must be G and the last letter must be H

$\underset{G}{1} \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot \underset{H}{1} = 720$

e) the arrangements must end in TH

$6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot \underset{T}{1} \cdot \underset{H}{1} = 720$

f) the arrangements begin and end in a vowel

$2 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 1 = 1440$

2 vowels to choose from — remaining vowel

8. The positions of captain and assistant captain have to be assigned for a school hockey team. If there are 15 players on the team, in how many ways can the captain and assistant captain be assigned? Explain your method. *(2 marks)*

players $\rightarrow {}_{15}P_2 \leftarrow$ positions $\qquad \frac{n!}{(n-r)!}$

15 players therefore 15 options $= \frac{15!}{(15-2)!} = \frac{15!}{13!} = \frac{15 \times 14 \times 13!}{13!}$

second spot could be filled by remaining 13 players $= 15 \times 14 = 210$

$\underline{15} \quad \underline{14} = 15 \times 14 = 210$

2 positions therefore two spaces

Notes

Assignment 1.2: Permutations and Combinations (continued)

4. How many permutations are there of the letters of the following words? *(3 × 2 marks each = 6 marks)*

a) HARDWARE

8 — 2 A's, 2 R's

$\frac{8!}{2!2!} = 10,080$

b) RIFFRAFF

8

2 R
4 F

$\frac{8!}{2!4!} = \frac{8 \times 7 \times 6 \times 5}{2} = 840$

c) MATHEMATICAL

12

2 M
3 A
2 T

$\frac{12!}{2!3!2!} = \frac{479,001,600}{24} = 19,958,400$

5. In how many ways can the letters of the word MAXIMUM be arranged if the three Ms must be together? Explain your process. *(2 marks)*

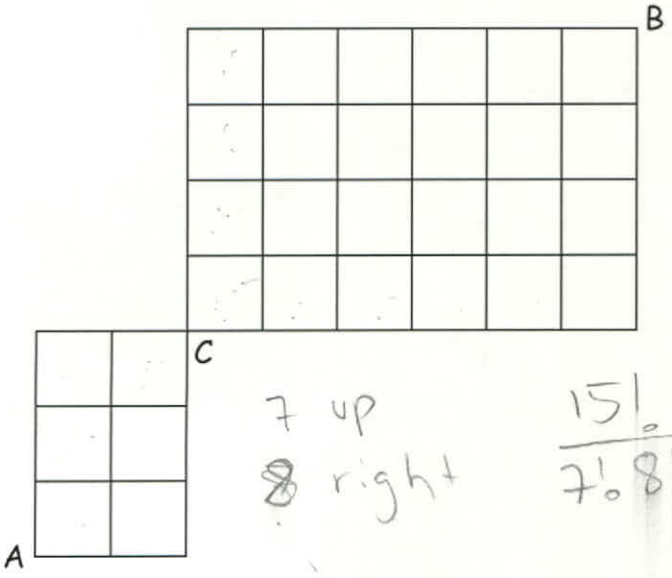
$\underline{1}_{m} \; \underline{1}_{m} \; \underline{1}_{m} \; \underline{4} \; \underline{3} \; \underline{2} \; \underline{1} = 24$

Order matters so as long as the 3 m's are together, it takes away 3 possible letters from the 7 letters. therefor 4 options are remaining, then 3, 2 and 1.

continued

Assignment 1.2: Permutations and Combinations (continued)

6. In how many different ways can person A walk to person B, passing through street intersection C, if the trip takes exactly 15 blocks? *(3 marks)*



7 up
8 right

$\frac{15!}{7!8!} = \frac{15 \times 14 \times 13 \times 12 \times 11 \times 10 \times 9 \times 8}{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1} = \frac{180180}{28}$

$= 6435$

7. Lotto 6-49 is a lottery in which a person selects six different numbers from 1 to 49. The order that the numbers are selected does not matter. How many ways can a selection be made? *(1 mark)*

$_{49}C_6 = 13,983,816$

8. On a Grade 12 Mathematics examination, students must answer exactly five of the first six questions and three out of the last five questions. In how many ways can this be done? *(2 marks)*

$_6C_5 \times {}_5C_3 = \frac{6!}{5!} \times \frac{5!}{2!3!} = 6 \times \frac{(5 \times 4 \times 3)}{3 \times 2 \times 1} = 6 \times \frac{60}{6} = 6 \times 10$

$= 60$

continued

Assignment 1.2: Permutations and Combinations (continued)

9. Using the digits 2, 2, 2, 3, 3, 4, 5, how many

   a) seven-digit numbers can be formed? *(2 marks)*

7 6 5 4 3 2 1 = 5040

$7! = 5040$

   b) seven-digit numbers can be formed if the number is greater than 3,400,000? *(5 marks)*

4 1 5 4 3 2 1 = 480

needs to start with 3 or higher
4 options

needs to start with 4 or higher
2 options but after the first one is chosen only 1 option remains

   c) seven-digit numbers can be formed if the number is even? *(3 marks)*

6 5 4 3 2 1 4 = 2880

4 options for even #

continued

Assignment 1.2: Permutations and Combinations (continued)

10. How many rectangles of various sizes are formed when seven vertical lines are intersected by four horizontal lines? *(2 marks)*

11. How many ways can a committee of five be chosen from 10 girls and eight boys so that the girls have a majority on the committee? *(4 marks)*

18 total / committee of 5 / 3 girls on the team

$_{10}C_3 \cdot {}_8C_2 = 120 \times 28 = 3360$ 2 boys

4 girls
1 boy

$_{10}C_4 \cdot {}_8C_1 = 210 \times 8 = 1680$

$3360 + 1680 = 5040$

12. Solve for n in the equation using the combination formula: ${}_{n+2}C_4 = 6({}_nC_2)$ *(4 marks)*

$nC_r = \frac{n!}{(n-r)!r!}$

$\frac{(n+2)!}{(n+2-4)!4!} = 6\left(\frac{n!}{(n-2)!2!}\right)$

$\frac{(n+2)!}{(n-2)!4!} = \frac{6n!}{(n-2)!2!}$

$\frac{(n+2)!(n-2)!}{(n-2)!(n)!} = \frac{6(4!)}{2!}$

$\frac{(n+2)!}{n!} = 6 \times 12$

$\frac{(n+2)! \times n!}{n!} = 72$

$\frac{n^2 + 2!}{2!} = \frac{72}{2}$

Assignment 1.3

The Binomial Theorem

Total: 24 marks

You need to show all your steps as you work out your solutions, and make sure your answers are clear (include units, where appropriate). Check the Introduction for the Marking Guidelines that will be used for all assignments and examinations.

1. Expand and simplify, using the Binomial Theorem. (2 × 4 *marks each* = *8 marks)*

   a) $(2x+1)^7$ $([2x]+1)^7$

$\binom{7}{0}(2x)^7\times 1^0+\binom{7}{1}\times(2x)^6\times 1^1+\binom{7}{2}\times(2x)^5\times 1^2+\binom{7}{3}\times(2x)^4\times 1^3+\binom{7}{4}(\times 2x)^3\times 1^4+\binom{7}{5}\times(2x)^2\times 1^5$
$+\binom{7}{6}\times(2x)^1\times 1^6+\binom{7}{7}\times(2x)^0\times 1^7$

$1\times 128x^7+7\times 64x^6+21\times 32x^5+35\times 16x^4+35\times 8x^3+21\times 4x^2+7\times 2x+1\times 1$

$128x^7+448x^6+672x^5+560x^4+280x^3+84x^2+14x+1$

   b) $\left(2+\frac{1}{x}\right)^3$

$\binom{3}{0}\times 2^3\times\left(\frac{1}{x}\right)^0+\binom{3}{1}\times 2^2\times\left(\frac{1}{x}\right)^1+\binom{3}{2}\times 2^1\times\left(\frac{1}{x}\right)^2+\binom{3}{3}\times 2\times\left(\frac{1}{x}\right)^3$

$1\times 8+3\times\frac{4}{x}+3\times\frac{2}{x^2}+1\times\frac{1}{x^3}$

$8+3\times\frac{4}{x}+3\times\frac{2}{x^2}+1\times\frac{1/3}{x}$

$8+\frac{12}{x}+\frac{6}{x^2}+\frac{1/3}{x}$

continued

Assignment 1.3: The Binomial Theorem (continued)

2. Write and simplify the first three terms of $\left(x-\frac{1}{x}\right)^{10}$. *(3 marks)*

$\binom{10}{0} \cdot x^{10} \cdot \left(-\frac{1}{x}\right)^0 + \binom{10}{1} \cdot x^9 \cdot \left(-\frac{1}{x}\right)^1 + \binom{10}{2} \cdot x^8 \cdot \left(-\frac{1}{x}\right)^2 + \binom{10}{3} \cdot x^7 \cdot \left(-\frac{1}{x}\right)^3 \ldots$

$1x^{10} + 10x\left(-x^8\right) + 45x^6 + 120x\left(-x^4\right) \ldots$

$x^{10} - 10x^8 + 45x^6 - 120x^4 \ldots$

3. Find the fifth term of $\left(\frac{y}{4}-\frac{2}{y}\right)^7$. *(2 marks)*

$\binom{7}{4} \cdot \left(\frac{y}{4}\right)^3 \cdot \left(-2y^{-1}\right)^4$

$= 35 \cdot \frac{1}{4^4}$

$= \frac{35}{4^4}$

continued

Assignment 1.3: The Binomial Theorem (continued)

4. Find and simplify.

a) The middle term of $\left(2x - \frac{1}{2x}\right)^{12}$. *(3 marks)*

7th term = middle.

$\binom{12}{6}(2x)^6\left(-\frac{1}{2x}\right)^6 = 924\left(36x^6\right)\left(\frac{1}{36x^6}\right) = 924$

b) The term containing x^{20} in $\left(2x - x^4\right)^{14}$. *(3 marks)*

$x^{14}, x^{15}, x^{16}, x^{17}, x^{18}, x^{19}, x^{20}$ ... 7th term

$\binom{14}{6}(2x)^6\left(-x^4\right)^6 = 3003\left(36x^6\right)\left(\frac{1}{256x^6}\right)$

$= 3003\left(\frac{36x^6}{256x^6}\right) = 3003\left(\frac{9}{64}\right) = \frac{27027}{64}$

c) The constant term in $\left(2x^4 - \frac{1}{2x^2}\right)^{12}$. *(3 marks)*

d) The fourth term in $\left(\frac{x^3}{3} - 2\right)^9$. *(2 marks)*

$\binom{9}{3}\left(\frac{x^3}{3}\right)^6\left(-2\right)^4 = 84\left(\frac{x^{18}}{3}\right)(16)$

$= 84\left(\frac{16x^{18}}{3}\right) = 448x^{18}$

Notes

Module 1 Summary

Congratulations, you have finished the first module in the course! In this module, you learned counting techniques that allowed you to count the number of unordered selections, called combinations, and the number of ordered selections, called permutations. It can be quite complex to count the number of arrangements of objects in certain situations, but now you know how to count these arrangements quickly and easily!

You also learned how to use the Binomial Theorem, including Pascal's Triangle, to expand binomials. This method of expanding binomials is widely used, as it decreases the amount of multiplying, adding, and simplifying you have to complete, which also decreases the number of mistakes you may make.

In the next module, you will be learning about function transformations.

Submitting Your Assignments

It is now time for you to submit Assignments 1.1 to 1.3 to the Distance Learning Unit so that you can receive some feedback on how you are doing in this course. Remember that you must submit all the assignments in this course before you can receive your credit.

Make sure you have completed all parts of your Module 1 assignments and organize your material in the following order:

- ❑ Module 1 Cover Sheet (found at the end of the course Introduction)
- ❑ Assignment 1.1: The Fundamental Counting Principle and Permutations
- ❑ Assignment 1.2: Permutations and Combinations
- ❑ Assignment 1.3: The Binomial Theorem

For instructions on submitting your assignments, refer to How to Submit Assignments in the course Introduction.

Notes

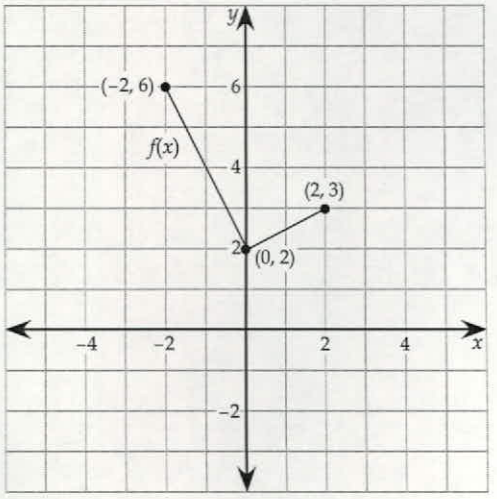
Assignment 2.1

Transformations of Functions

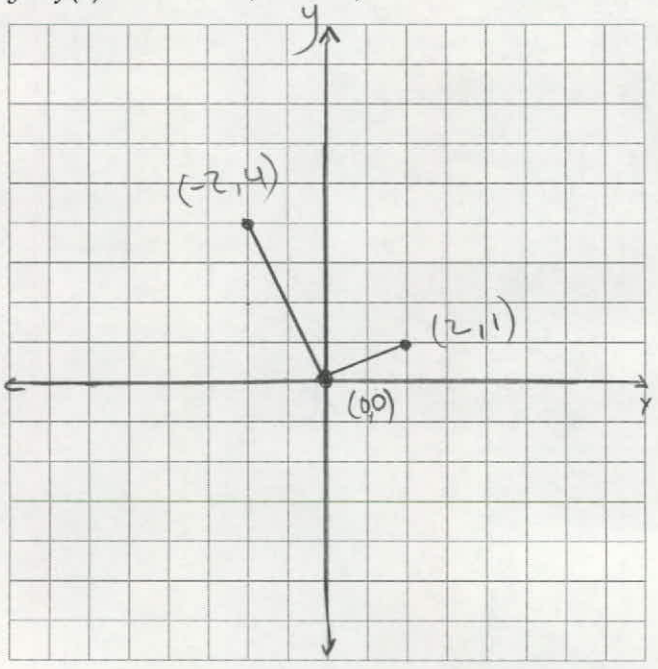
Total: 28 marks

You need to show all your steps as you work out your solutions, and make sure your answers are clear (include units, where appropriate). Check the Introduction for the Marking Guidelines that will be used for all assignments and examinations. **Note:** Be sure to label your scale and key points in your transformations.

1. Given the sketch of $f(x)$ shown below, sketch each of the following functions. State the domain and range of each function.



a) $y = f(x) - 2$ *(2 marks)*



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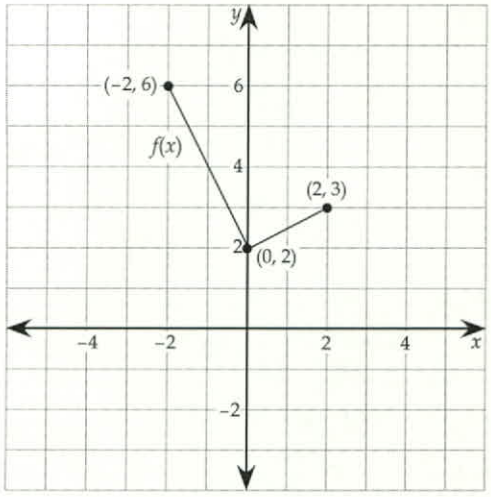
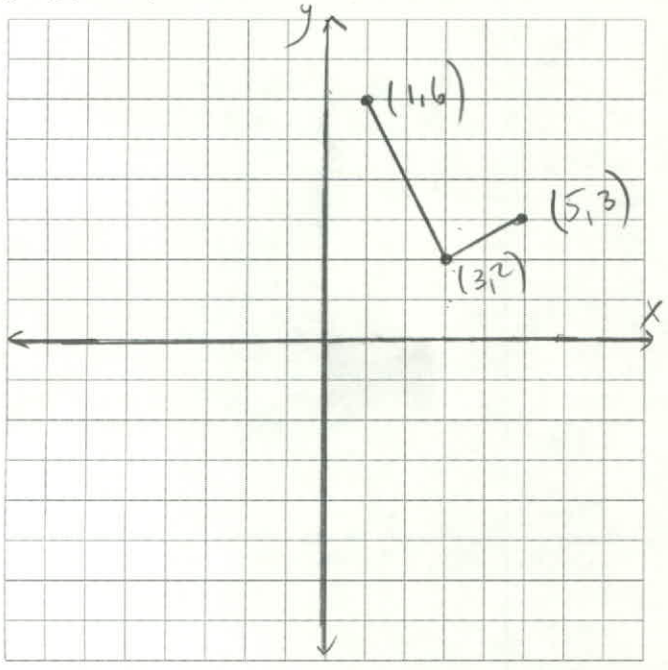


b) $y = f(x - 3)$ *(2 marks)*



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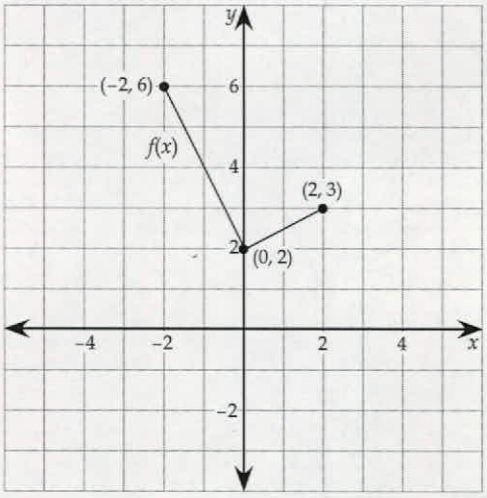
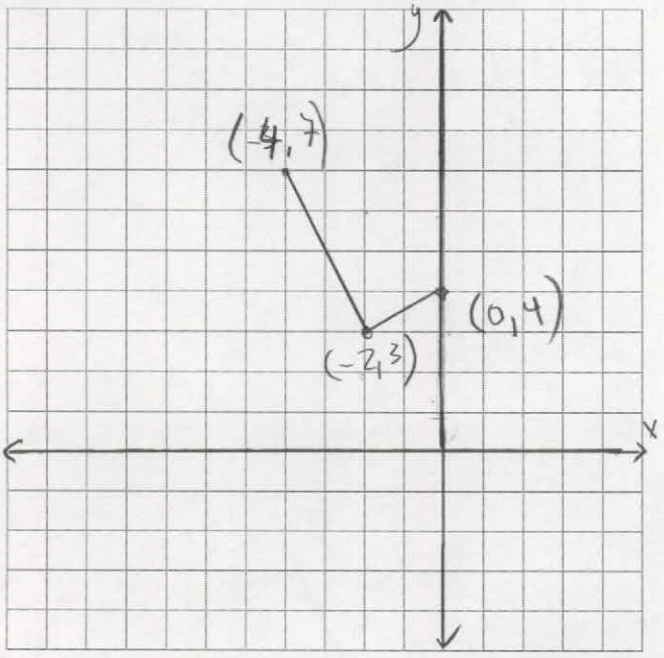


c) $y = f(x + 2) + 1$ *(3 marks)*

2 left

1 up



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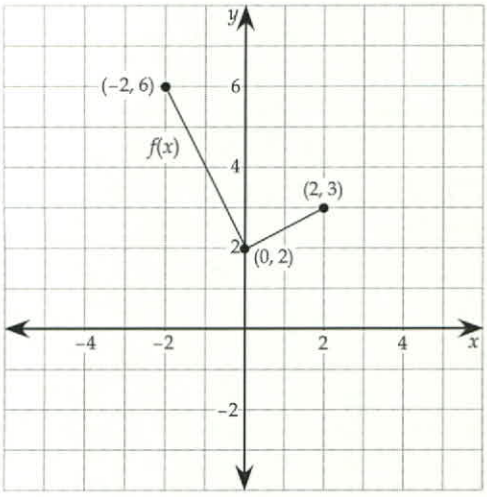
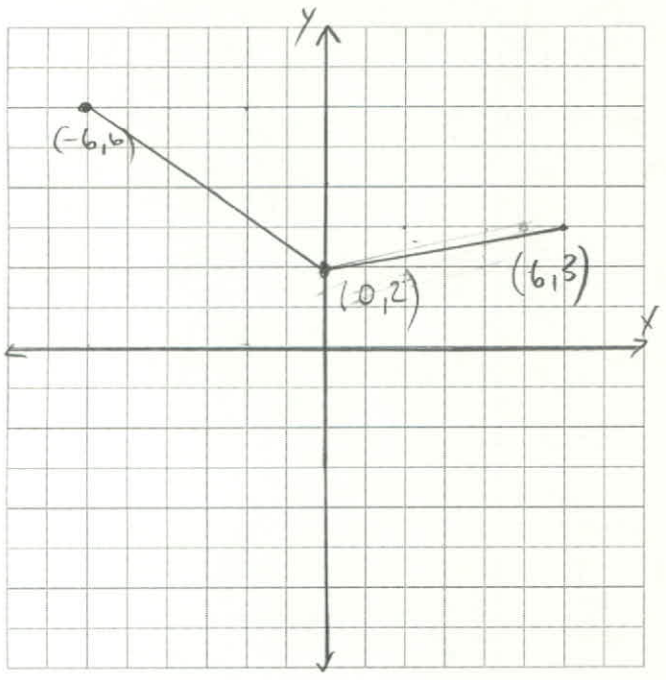


d) $y = f\left(\frac{1}{3}x\right)$ *(2 marks)*

times x values by 3



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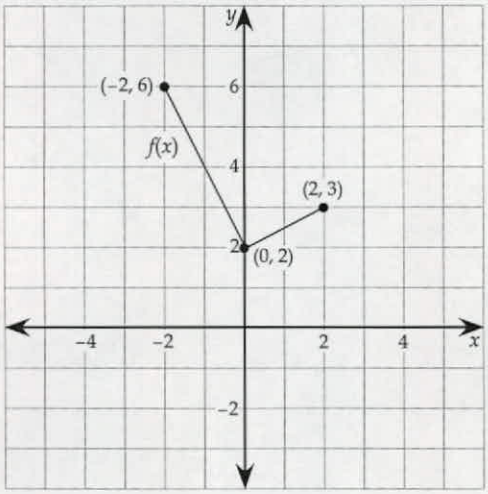


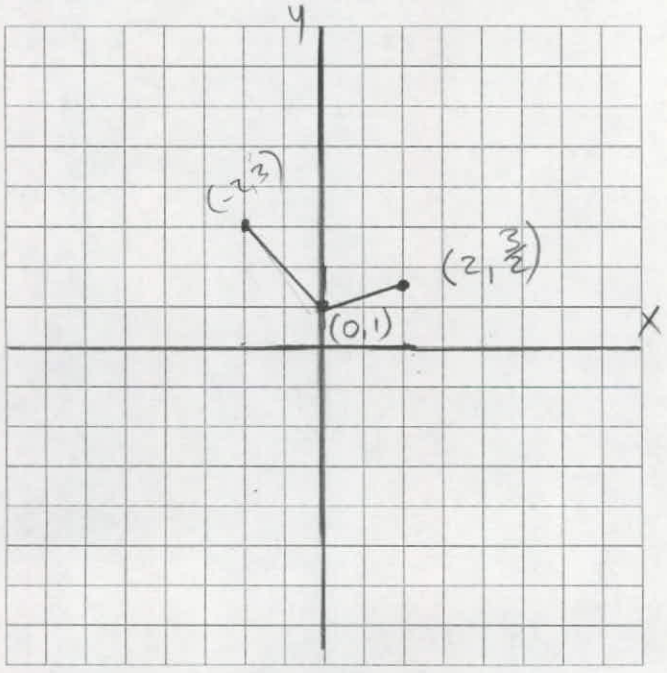
multiply y's by $\frac{1}{2}$

e) $y = \frac{1}{2} f(x)$ *(2 marks)*

$\frac{6}{1} \times \frac{1}{2} = \frac{6}{2} = 3$

$3 \times \frac{1}{2} = \frac{3}{2}$

$2\times$



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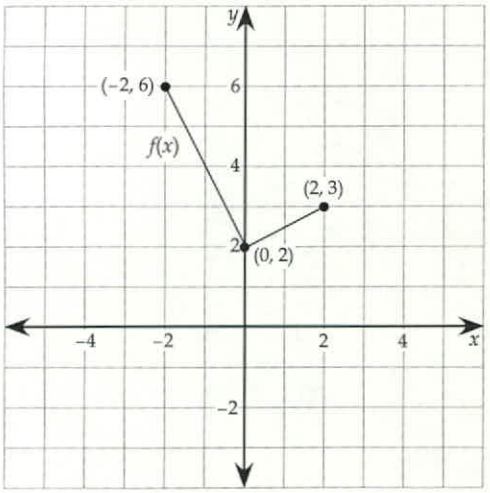


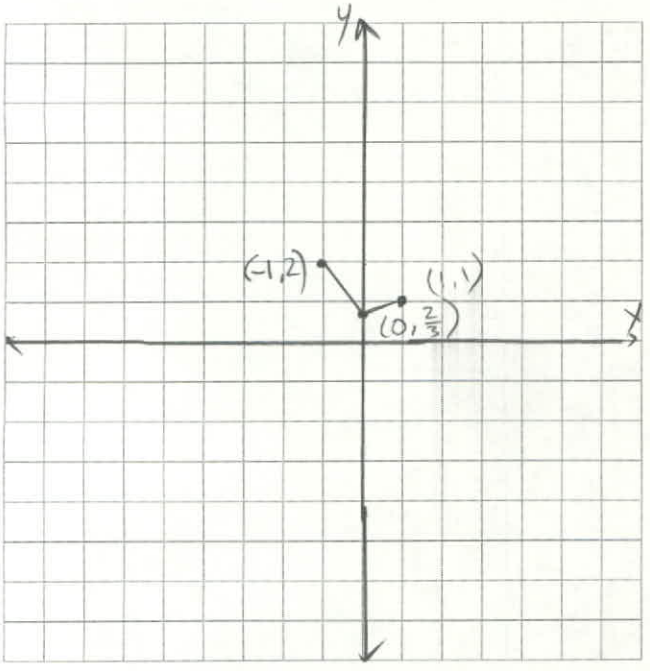
multiply y's by 1/3

$6 \times \frac{1}{3} = 2$

$2 \times \frac{1}{3} = \frac{2}{3}$

$3 \times \frac{1}{3}$ 1

f) $y = \frac{1}{3} f(2x)$ *(3 marks)*



divide x's by 2

$-2 \div 2 = -1$

$0 \div 2 = 0$

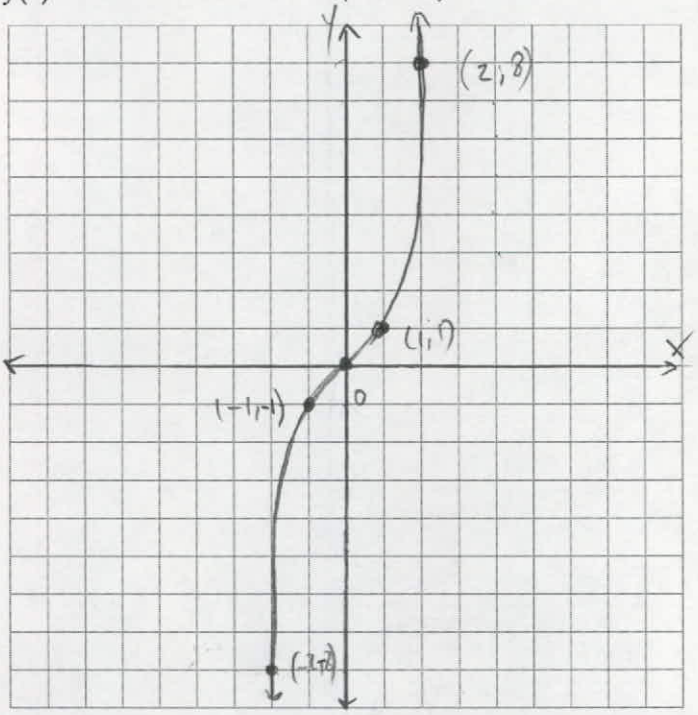
$2 \div 2 = 1$

continued

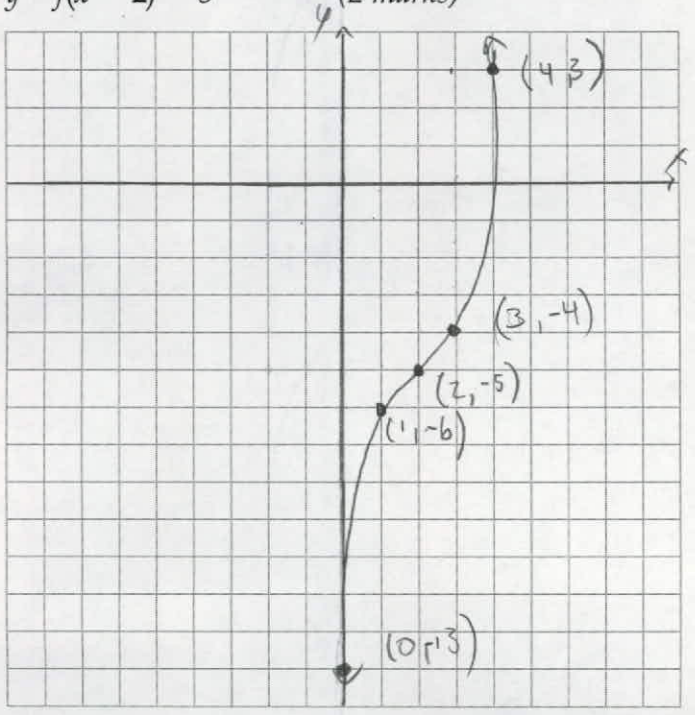
Assignment 2.1: Transformations of Functions (continued)

2. Let $f(x) = x^3$. Sketch each of the following functions.

a) $f(x)$ *(1 mark)*



b) $y = f(x - 2) - 5$ *(2 marks)*

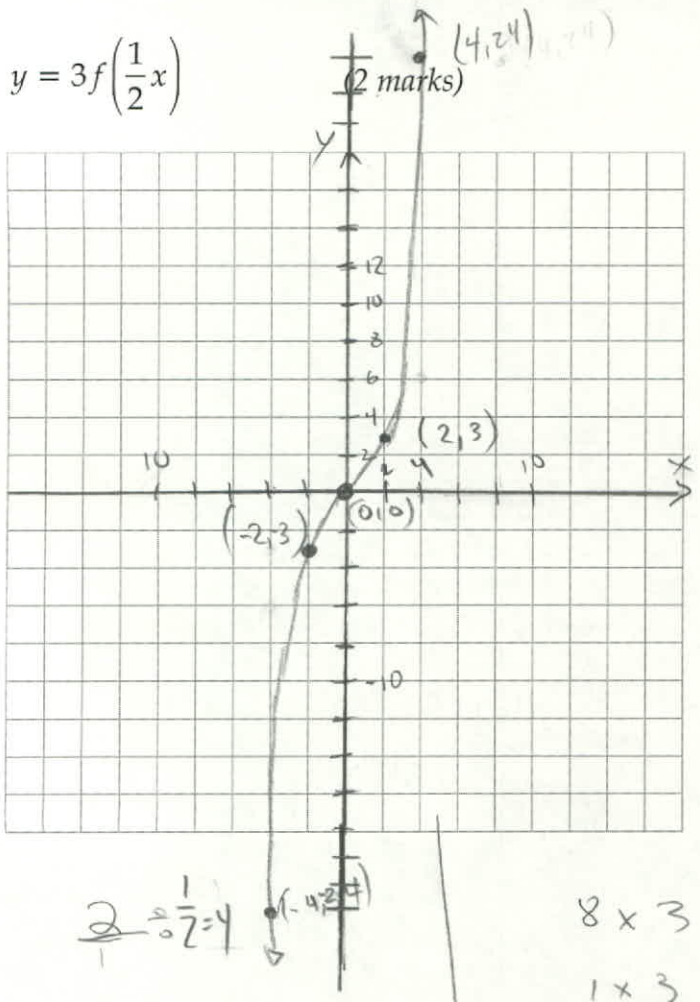


2 right
5 down

continued

Assignment 2.1: Transformations of Functions (continued)

c) $y = 3f\left(\frac{1}{2}x\right)$ (2 marks)



divide the x's by $\frac{1}{2}$

multiply y's by 3

x	y
(2,8)	(4,24)
(1,1)	(2,3)
(0,0)	(0,0)
(-1,-1)	(-2,-3)
(-2,-8)	(-4,-24)

$2 \div \frac{1}{2} = 4$

$1 \div \frac{1}{2} = 2$

$0 \div \frac{1}{2} = 0$

$-1 \div \frac{1}{2} = -2$

$-2 \div \frac{1}{2} = -4$

X

$8 \times 3 = 24$

$1 \times 3 = 3$

$0 \times 3 = 0$

$-1 \times 3 = -3$

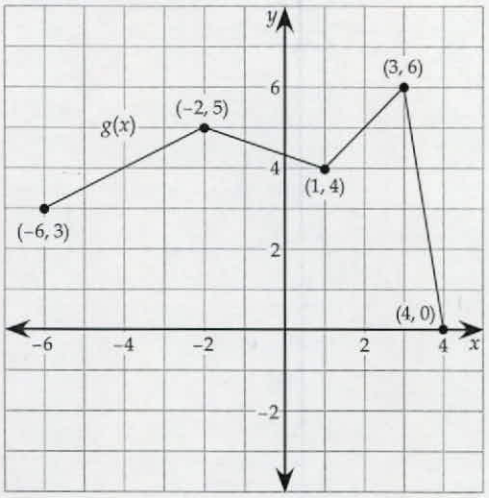
$-8 \times 3 = -24$

Y

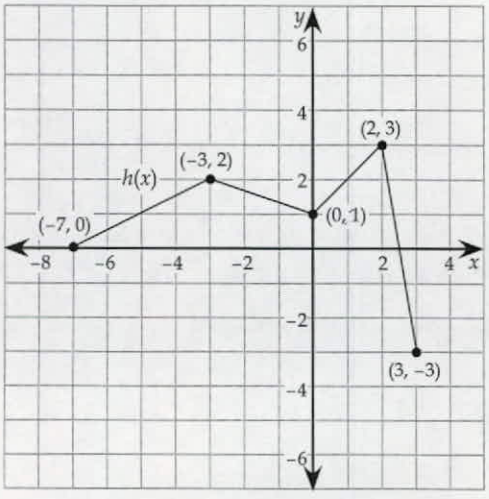
continued

Assignment 2.1: Transformations of Functions (continued)

3. Each of graphs below represents a translation, stretch, and/or compression of the given function, $g(x)$, shown below. Write an equation for each new function in terms of $g(x)$ using correct notation.



a) *(1 mark)*



Y
:3

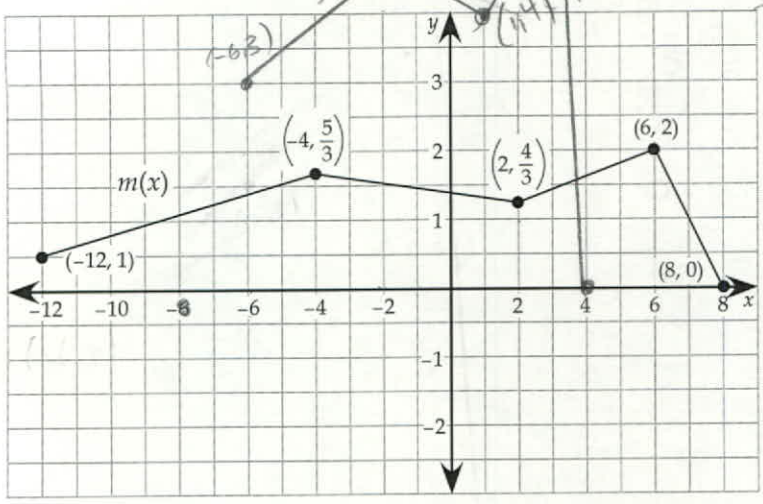
-1 = left 1

↓ down 3
3

$g(x) = f(x+1) - 3$

continued

b) *(2 marks)*



original → this graph

$(-6, 3) = (-12, 1)$

$(-2, 5) = (-4, \frac{5}{3})$

$(1, 4) = (2, \frac{4}{3})$

$(3, 6) = (6, 2)$

$(4, 0) = (8, 0)$

x values: × 2

y values: ÷ by 3

$j(x) = \frac{1}{3} g\left(\frac{1}{2}x\right)$

4. How should the function equation for $f(x)$ be modified if you want to perform each of the following transformations? *(3 × 2 marks each = 6 marks)*

a) translate the graph two units to the right and three units up

$f(x - 2) + 3$

b) compress the graph vertically by a factor of 5 and horizontally by a factor of 3

$\frac{1}{5} f\left(\frac{1}{3}x\right)$

c) stretch the graph vertically by a factor of four and translate it three units to the left

$\frac{1}{4} f(3x)$

Assignment 2.2

Combinations of Transformations

Total: 20 marks

You need to show all your steps as you work out your solutions, and make sure your answers are clear (include units, where appropriate). Check the Introduction for the Marking Guidelines that will be used for all assignments and examinations.

1. Assume a graph of $f(x)$ is given. Describe how the graph of $F(x)$ is obtained, using the correct order of transformations. *(2 × 2 marks each = 4 marks)*

   a) $F(x) = 3\left(f\left(\frac{1}{3}(x-1)\right)\right)+7$

   F(x) is a vertical stretch by a factor of 3 and a horizontal compression by a factor of $\frac{1}{3}$ which is then translated 1 unit to the right an 7 units up

   b) $F(x) = 7\left(f\left(3(x+2)\right)\right)-5$

   F(x) is a vertical stretch by a factor of 7 and a horizontal stretch by a factor of 3 which is then translated 2 units to the left an 5 units down

2. Given the function $y = f(x)$, write the equation of the form $y = af(b(x - h)) + k$ that would result from each combination of transformations. *(2 × 2 marks each = 4 marks)*

   a) Horizontal translation 3 units to the left, vertical translation 4 units down, horizontal compression by a factor of 2, and a vertical stretch by a factor of $\frac{1}{4}$

   $y = af\left(b(x-h)\right)$

   $y = \frac{1}{4}f\left(\frac{1}{2}(x+3)\right)-4$

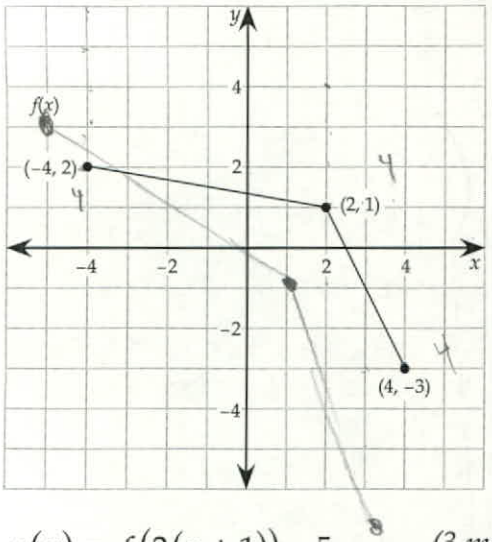
   b) Vertical compression by a factor of 6, horizontal stretch by a factor of 2, vertical translation 2 units up, and a horizontal translation 7 units to the right

   $y = \frac{1}{6}\left(f\left(2(x-7)\right)\right)+2$

continued

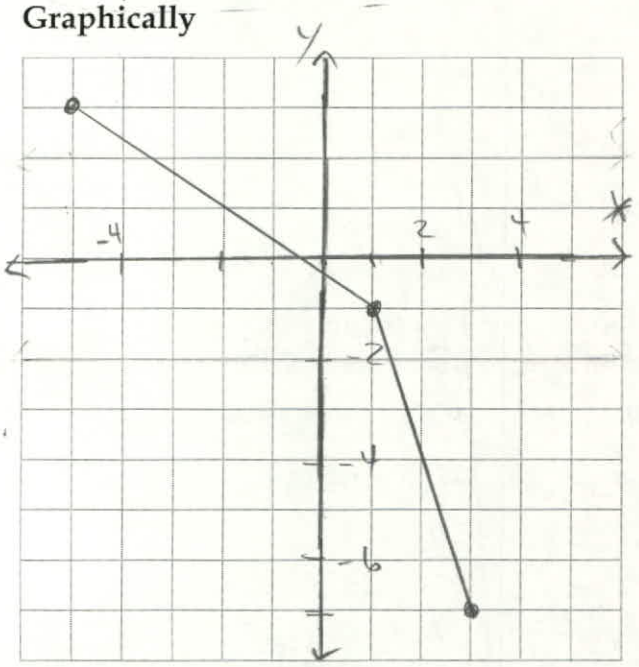
Assignment 2.2: Combinations of Transformations (continued)

3. Given the graph of $f(x)$ below, show each transformation algebraically and graphically.



a) $g(x) = f(2(x+1)) - 5$ *(3 marks)*

Graphically



Algebraically

horizontal stretch by 2
vertical stretch by 0
1 unit to the left
5 units down.

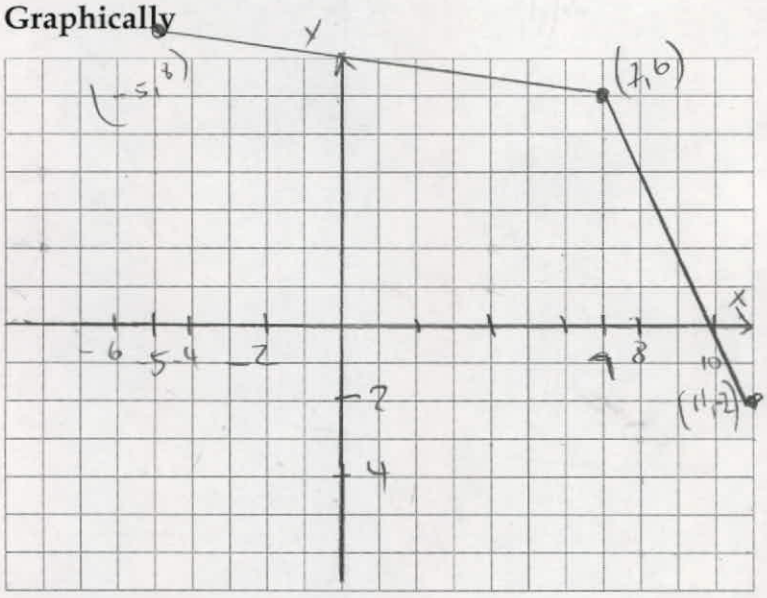
(−4, 8)

continued

Assignment 2.2: Combinations of Transformations (continued)

b) $h(x) = 2f\left(\frac{1}{2}(x-3)\right) + 4$ *(3 marks)*

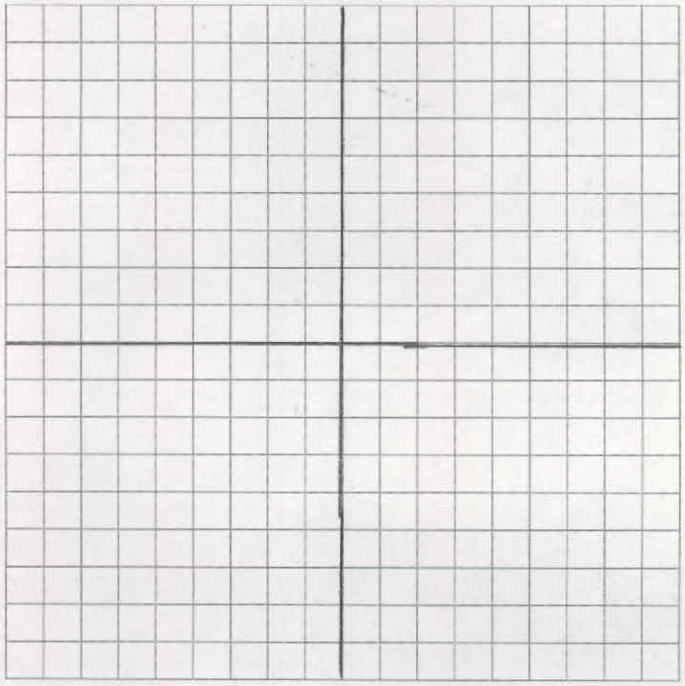
Graphically



Algebraically

- horizontal stretch by $\frac{1}{2}$
- vertical stretch by 2
- translated into 3 units right
- and 4 units up

4. Sketch the function $f(x) = 2\left(\frac{1}{2}(x+3)\right)^2 - 5$ and state the domain and range of the function. *(4 marks)*

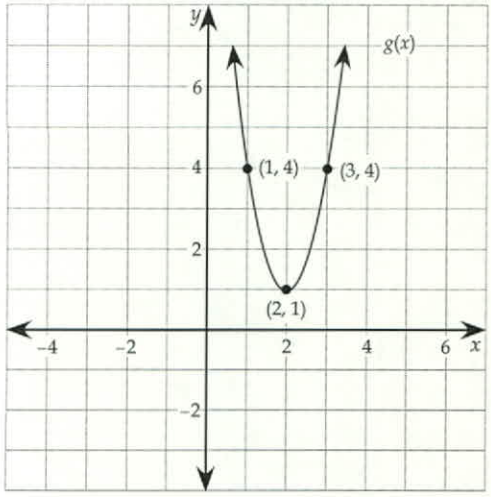


horizontal stretch by 2

continued

Assignment 2.2: Combinations of Transformations (continued)

5. The following graph represents a transformation of $f(x) = x^2$. Write an equation for the new function in terms of $f(x)$ using function notation. *(2 marks)*



Assignment 2.3

Operations on and Compositions of Functions

Total: 54 marks

You need to show all your steps as you work out your solutions, and make sure your answers are clear (include units, where appropriate). Check the Introduction for the Marking Guidelines that will be used for all assignments and examinations.

1. Using the functions $f(x) = x^2 + 3x - 18$, $g(x) = -x + 7$, and $h(x) = -x + 3$, determine a simplified function equation for the following and evaluate as indicated.

   a) $(f + g)(x)$ *(2 marks)*

   $(f+g)(x) = x^2 + 3x - 18 - x + 7$

   $= x^2 + 2x - 11$

   b) $(f + g)(3)$ *(1 mark)*

   $= x^2 + 3x - 18 - x + 7$

   $= (3)^2 + 3(3) - 18 - 3 + 7$

   $= 9 + 9 - 18 - 3 + 7 = 4$

   c) $\left(\frac{f}{h}\right)(x)$ *(2 marks)*

   $\frac{x^2 + 3x - 18}{-x + 3} = \frac{x^2 + 3x - 9}{-x}$

continued

Assignment 2.3: Operations on and Compositions of Functions (continued)

d) $\left(\frac{f}{h}\right)(2)$ *(1 mark)*

$\left(\frac{f}{h}\right)(x) = \frac{x^2 + 3x - 18}{-x + 3} = \frac{(2)^2 + 3(2) - 18}{-(2) + 3} = \frac{4 + 6 - 18}{1}$

$= -8$

e) $(h \cdot g)(x)$ *(2 marks)*

$= (-x + 3)(-x + 7)$

$= x^2 - 3x - 7x + 21$

$= x^2 - 10x + 21$

f) $(h \cdot g)(0)$ *(1 mark)*

$= (-x + 3)(-x + 7)$

$= (-0 + 3)(-0 + 7)$

$= 0 - 0 - 0 + 21$

$= 21$

continued

Assignment 2.3: Operations on and Compositions of Functions (continued)

g) $(h - g)(x)$ *(2 marks)*

$= -x + 3 + x - 7$

$= -4$

h) $(h - g)(-1)$ *(1 mark)*

$= -x + 3 + x - 7$

$= 1 + 3 - 1 - 7$

$= -4$

continued

Assignment 2.3: Operations on and Compositions of Functions (continued)

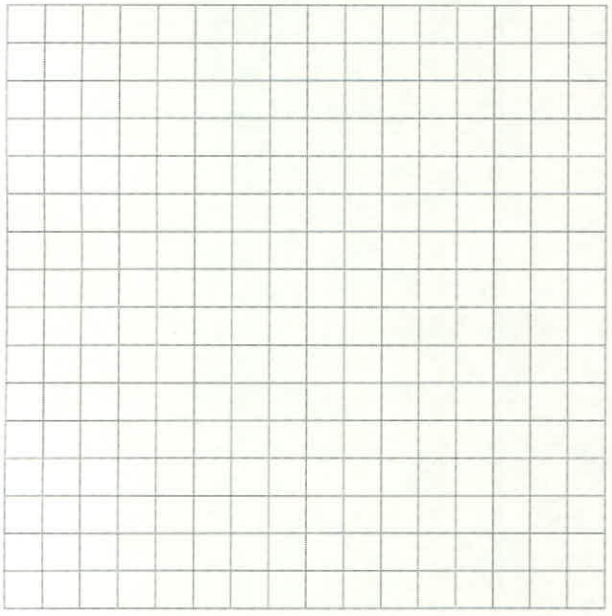
2. Let $f(x) = 2x^2 + 5x - 6$ and $g(x) = x^2 - 2x + 4$.

   a) Find $(f + g)(x)$. *(2 marks)*

$= 2x^2 + 5x - 6 + x^2 - 2x + 4$

$= 3x^2 + 3x - 2$

   b) Graph $(f + g)(x)$. Show your work. *(2 marks)*

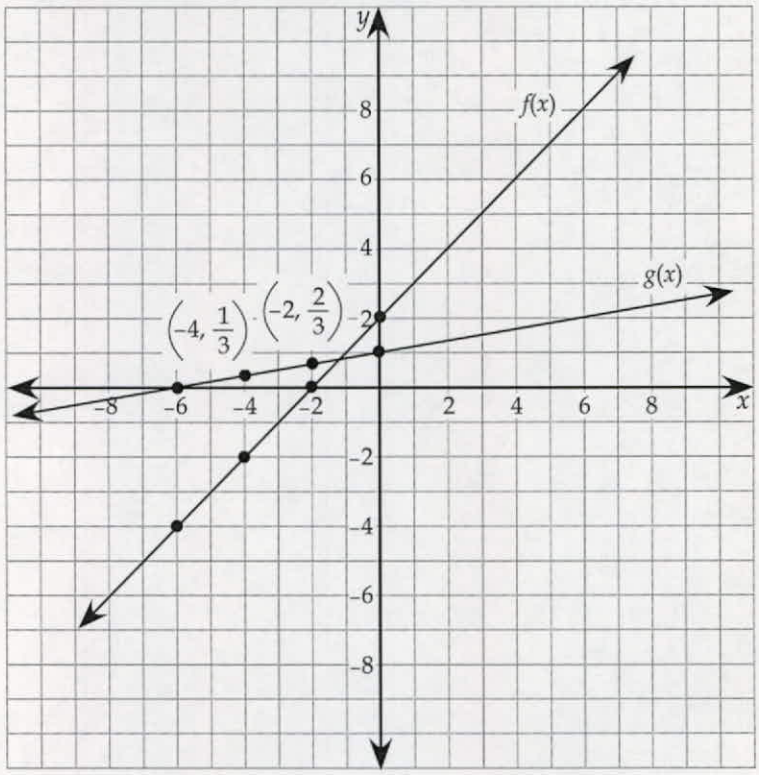


   c) State the domain and range. *(1 mark)*

continued

Assignment 2.3: Operations on and Compositions of Functions (continued)

3. Consider the graphs of $f(x)$ and $g(x)$ shown below.

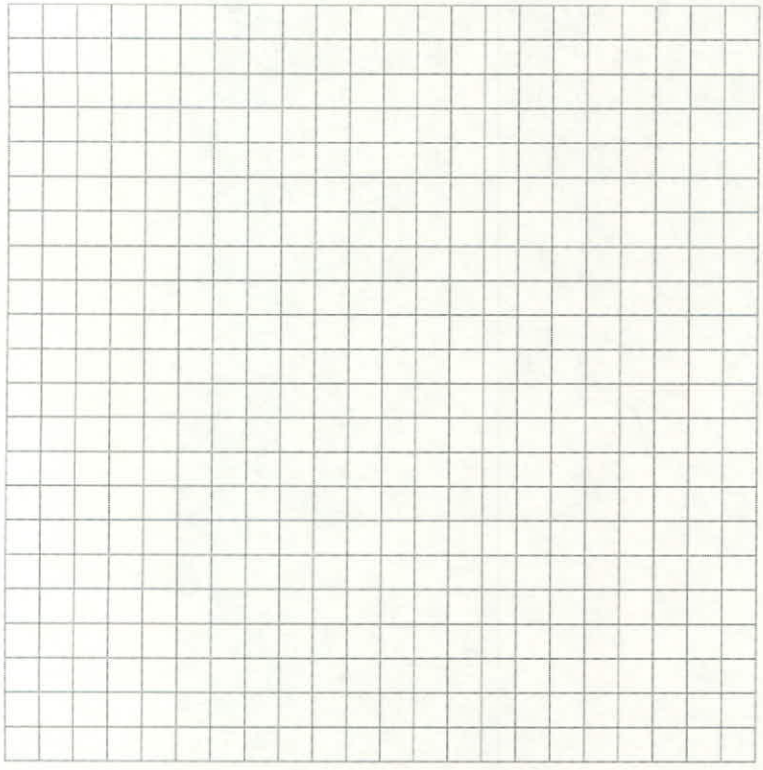


a) Create a table of values for $(f \cdot g)(x)$. *(2 marks)*

continued

Assignment 2.3: Operations on and Compositions of Functions (continued)

b) Sketch the graph of $(f \cdot g)(x)$. *(1 mark)*

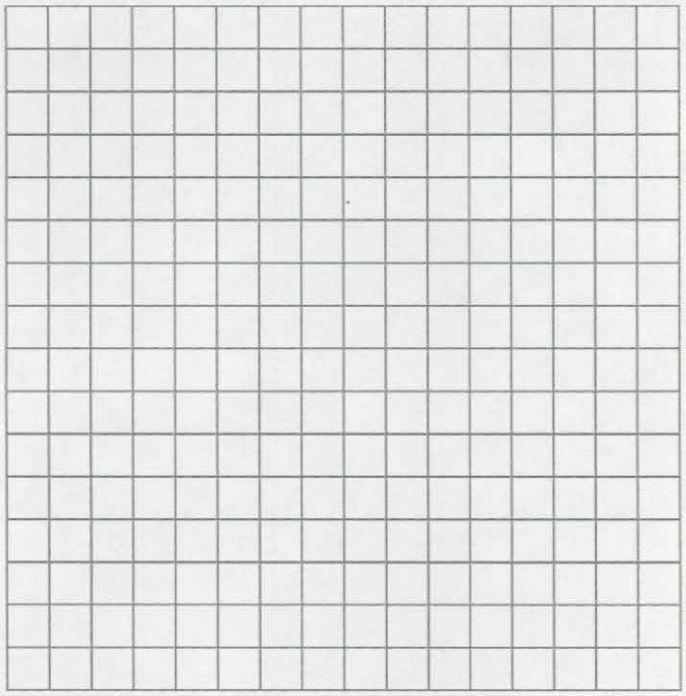


continued

Assignment 2.3: Operations on and Compositions of Functions (continued)

4. Let $f(x) = (2x + 4)(x + 1)$ and $g(x) = 2$. Determine the simplified equation $\left(\frac{f}{g}\right)(x)$.

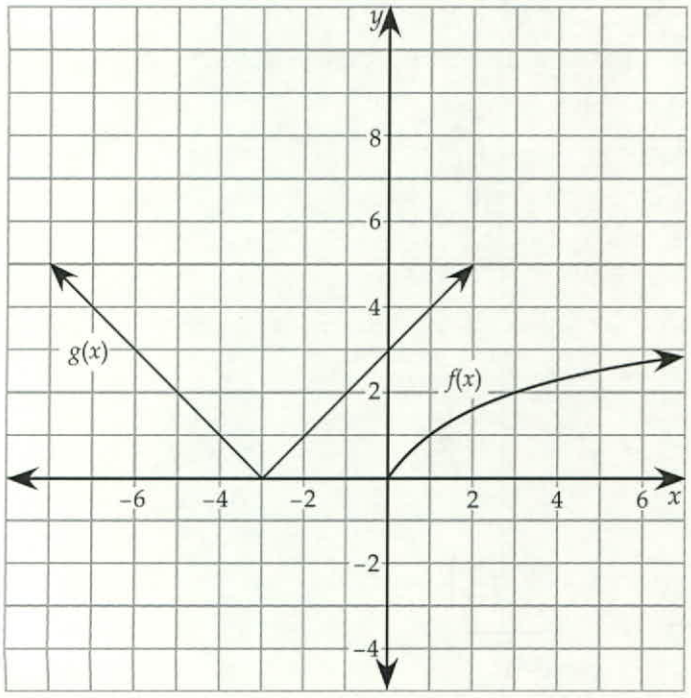
   Graph $\left(\frac{f}{g}\right)(x)$, stating the domain and range. *(4 marks)*



continued

Assignment 2.3: Operations on and Compositions of Functions (continued)

5. Use the graphs of $f(x)$ and $g(x)$ shown below to evaluate the following. *(4 × 1 mark each = 4 marks)*



a) $(g + f)(-3)$

b) $(f - g)(4)$

continued

Assignment 2.3: Operations on and Compositions of Functions (continued)

c) $(f \cdot g)(0)$

d) $\left(\frac{f}{g}\right)(1)$

6. Given that $t(x) = 2x - 6$ and $s(x) = \frac{1}{2}x + 3$, find the following.

a) $s(t(x))$ *(2 marks)*

b) $t(s(x))$ *(2 marks)*

c) $s(s(x))$ *(2 marks)*

continued

Assignment 2.3: Operations on and Compositions of Functions (continued)

d) $t(t(x))$ *(2 marks)*

e) $s(t(7))$ *(1 mark)*

f) $s(s(-3))$ *(1 mark)*

g) $t(s(8))$ *(1 mark)*

h) $t\left(t\left(-\frac{3}{4}\right)\right)$ *(1 mark)*

i) Does $s(t(x)) = t(s(x))$ for all values of x? Justify your answer using composition of functions. *(1 mark)*

continued

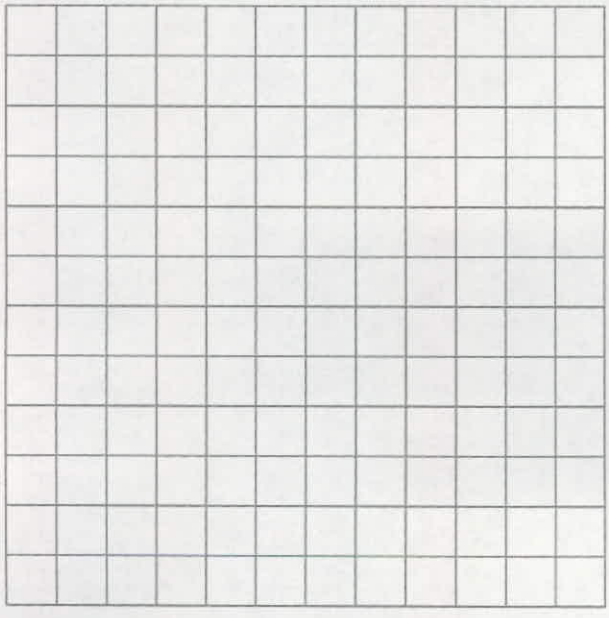
Assignment 2.3: Operations on and Compositions of Functions (continued)

7. If $f(x) = \sqrt{x+3}$ and $g(x) = x^2 - 1$,

   a) determine the simplified equation of $h(x) = g(f(x))$. *(2 marks)*

   b) determine the domain and range of $h(x) = g(f(x))$. *(2 marks)*

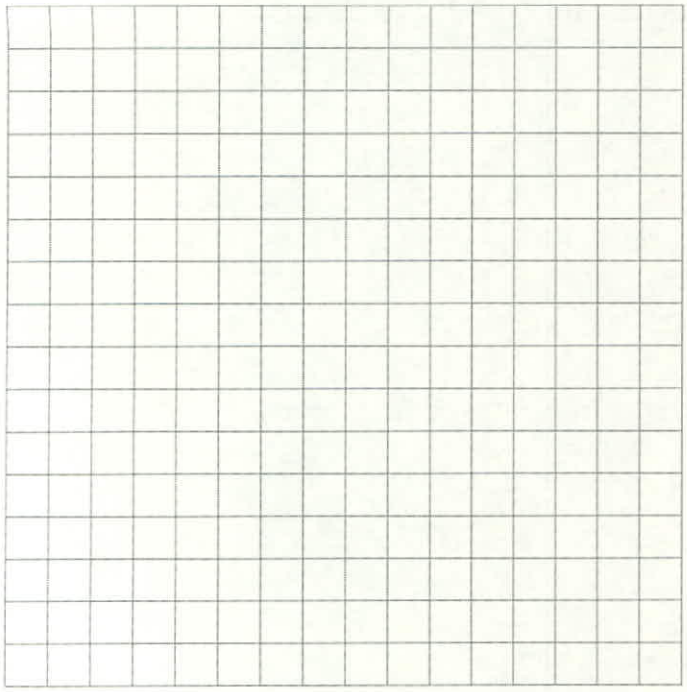
   c) sketch the graph of $h(x) = g(f(x))$. *(1 mark)*



continued

Assignment 2.3: Operations on and Compositions of Functions (continued)

8. If $f(x) = \dfrac{1}{x+2}$ and $g(x) = |x|$, determine the equation and sketch $g(f(x))$. *(4 marks)*



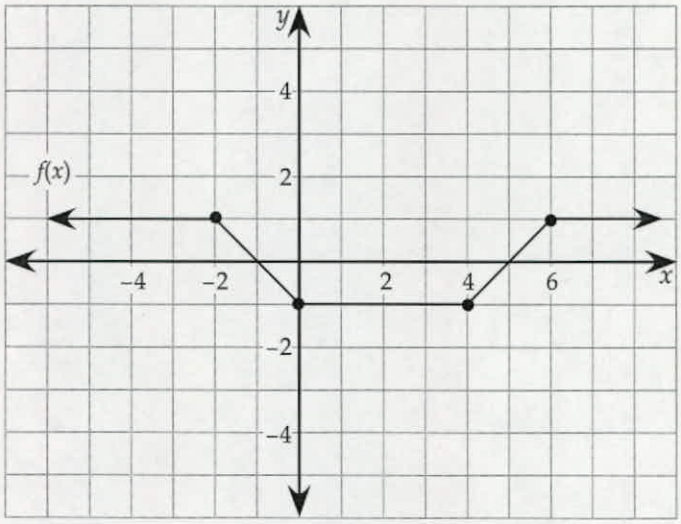
continued

Assignment 2.3: Operations on and Compositions of Functions (continued)

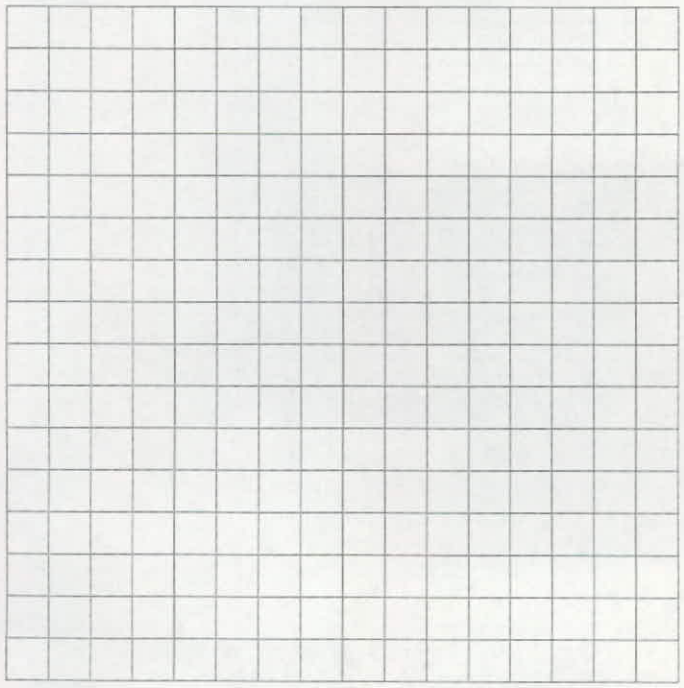
9. Use the graph of the function $f(x)$ shown below to sketch:

   a) $|f(x)|$ *(1 mark)*

   b) $\dfrac{1}{f(x)}$ *(3 marks)*



a) Graph of $|f(x)|$



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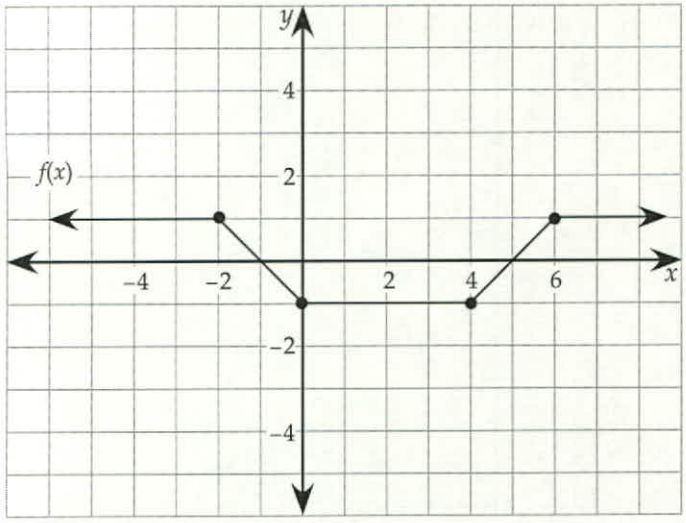
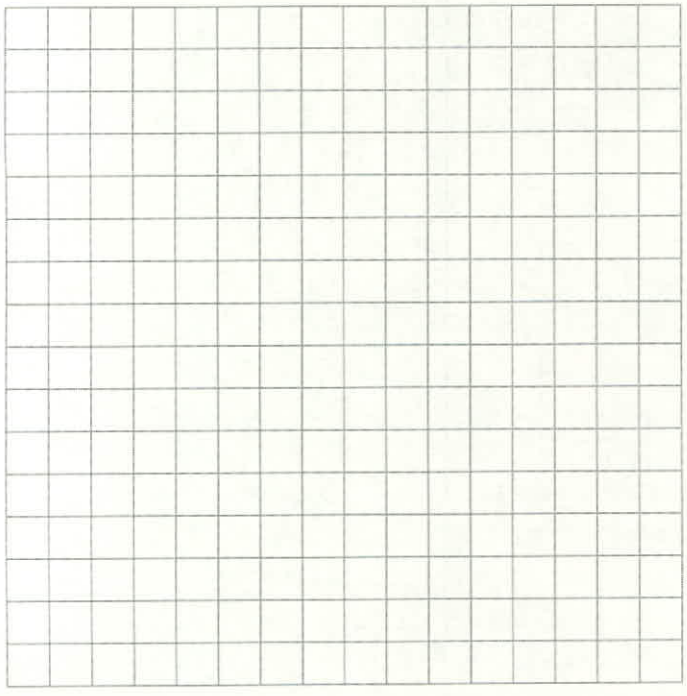
Assignment 2.3: Operations on and Compositions of Functions (continued)



b) Graph of $\frac{1}{f(x)}$



Module 2 Summary

Throughout this module, you expanded on your knowledge of functions. You learned how to transform functions with the use of translations, stretches, and compressions, similar to how you transformed quadratic functions in Grade 11 Pre-Calculus Mathematics. You also learned how to graph the reciprocal or the absolute value of any function when you were given the graph of that function. These strategies will come in handy throughout the rest of the course as a great deal of graphing is required. These strategies will also prepare you for learning about reflections in the next module.

You also learned about operations on functions and compositions of functions. Operations and compositions are different ways of combining functions. You may not notice it, but you will often encounter operations on functions and compositions of functions when you are dealing with more complex functions.

Submitting Your Assignments

It is now time for you to submit Assignments 2.1 to 2.3 to the Distance Learning Unit so that you can receive some feedback on how you are doing in this course. Remember that you must submit all the assignments in this course before you can receive your credit.

Make sure you have completed all parts of your Module 2 assignments and organize your material in the following order:

- ❑ Module 2 Cover Sheet (found at the end of the course Introduction)
- ❑ Assignment 2.1: Transformations of Functions
- ❑ Assignment 2.2: Combinations of Transformations
- ❑ Assignment 2.3: Operations on and Compositions of Functions

For instructions on submitting your assignments, refer to How to Submit Assignments in the course Introduction.

Notes

Assignment 3.1

Reflections in the *x*-axis and in the *y*-axis

Total: 34 marks

You need to show all your steps as you work out your solutions, and make sure your answers are clear (include units, where appropriate). Check the Introduction for the Marking Guidelines that will be used for all assignments and examinations.

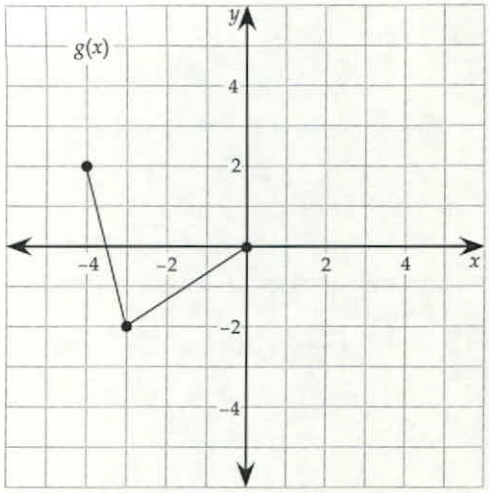
1. A function contains the ordered pairs (1, 0), (4, −3), (−6, 1), and (0, −5).
   a) What are the corresponding ordered pairs if this function is reflected through the *x*-axis (vertically)? *(1 mark)*

   b) What are the corresponding ordered pairs if this function is reflected through the *y*-axis (horizontally)? *(1 mark)*

continued

Assignment 3.1: Reflections in the x-axis and in the y-axis (continued)

2. Using the sketch of $g(x)$, sketch the following. Explain the transformations, in words or algebraically.



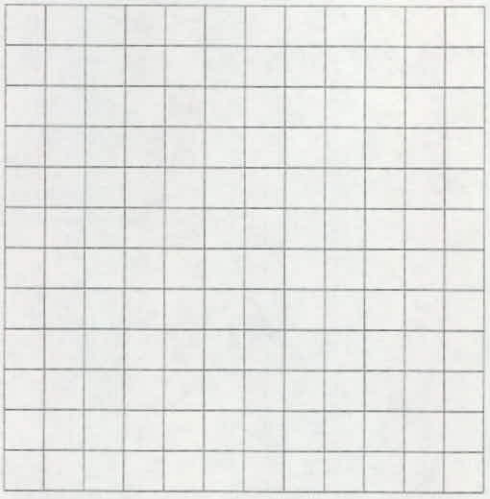
a) $y = g(-x) + 3$ *(2 marks)*

b) $y = -g(-(x - 1))$ *(3 marks)*

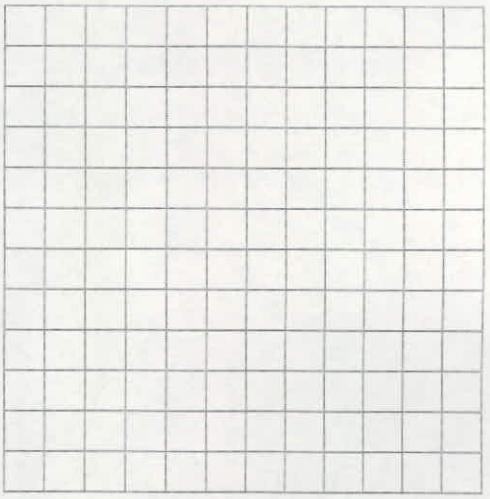
continued

Assignment 3.1: Reflections in the x-axis and in the y-axis (continued)

c) $y = -\frac{1}{2}g(x) + 3$ *(3 marks)*



d) $y = -g(-2(x+2))$ *(3 marks)*

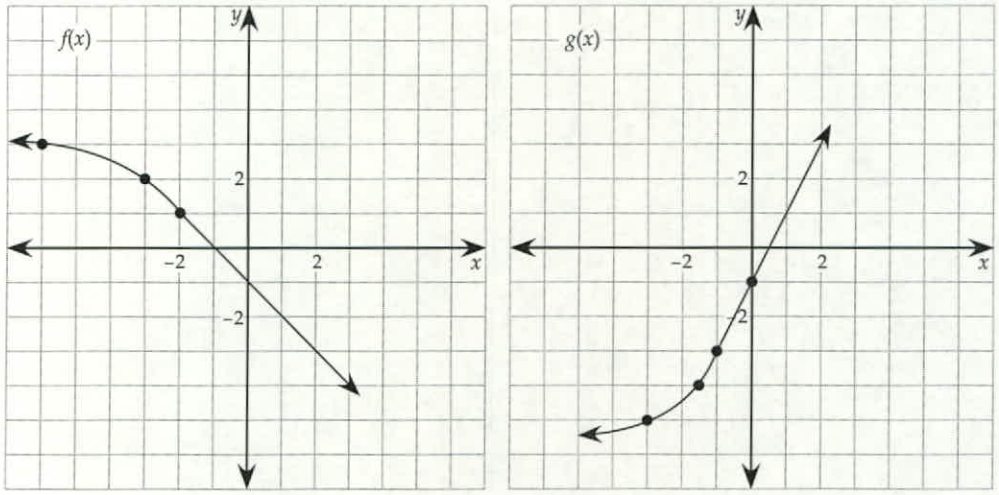


continued

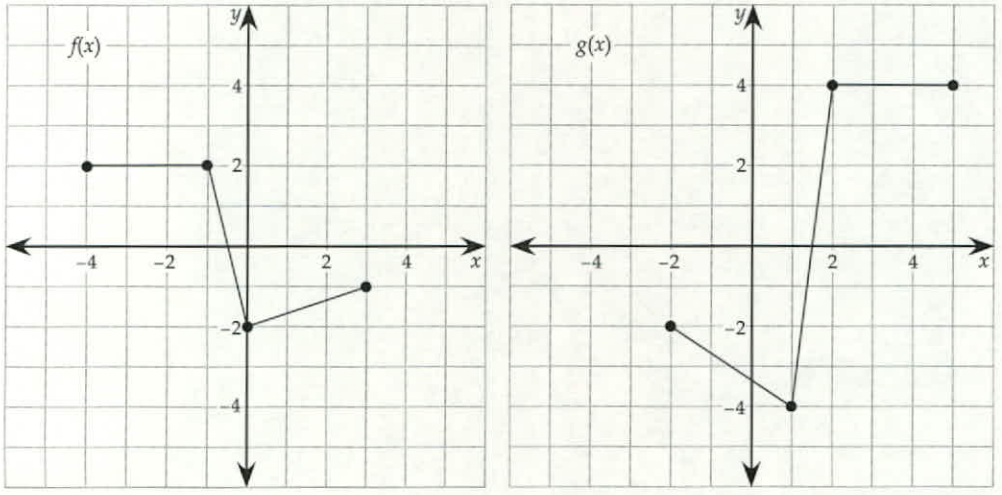
Assignment 3.1: Reflections in the x-axis and in the y-axis (continued)

3. For each of the graphs of $f(x)$ below, write the equation of the transformed function, $g(x)$, in terms of $f(x)$.

   a) *(2 marks)*



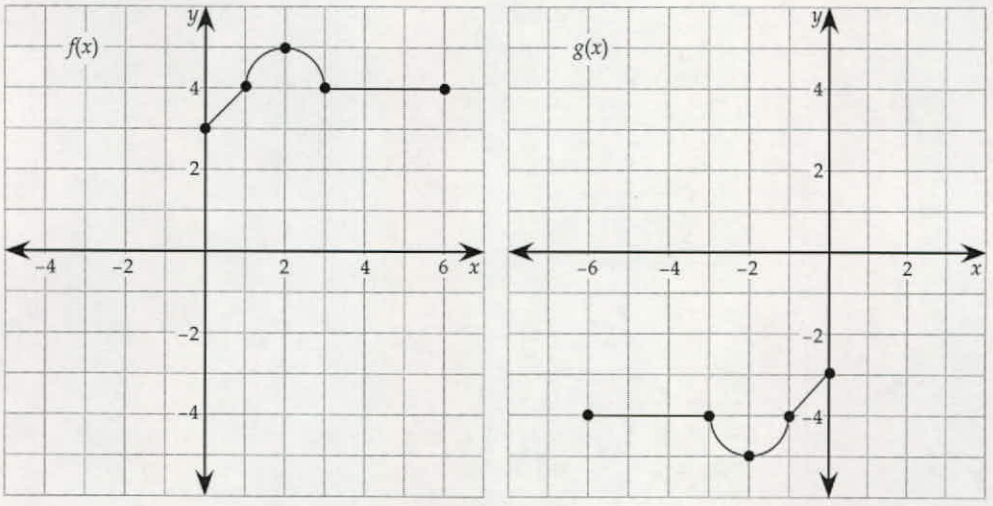
   b) *(2 marks)*



continued

Assignment 3.1: Reflections in the x-axis and in the y-axis (continued)

c) *(1 mark)*

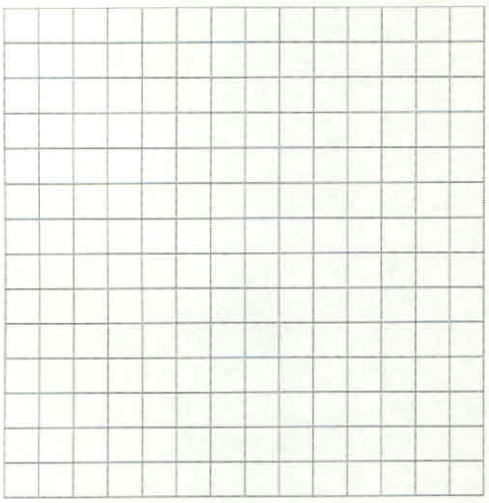


continued

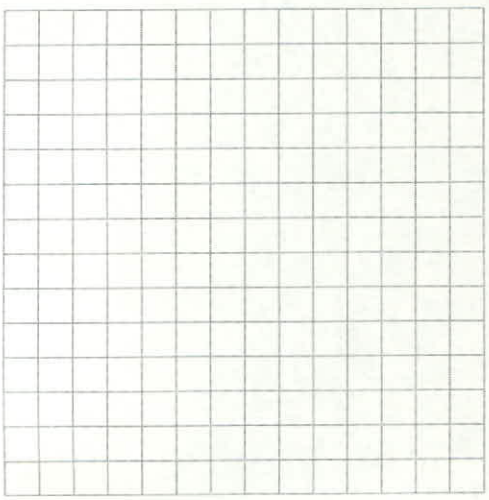
Assignment 3.1: Reflections in the x-axis and in the y-axis (continued)

4. Sketch the following functions.

   a) $f(x) = -|-(x+1)| - 3$ *(3 marks)*



   b) $g(x) = -3\sqrt{-\frac{1}{2}x}$ *(4 marks)*

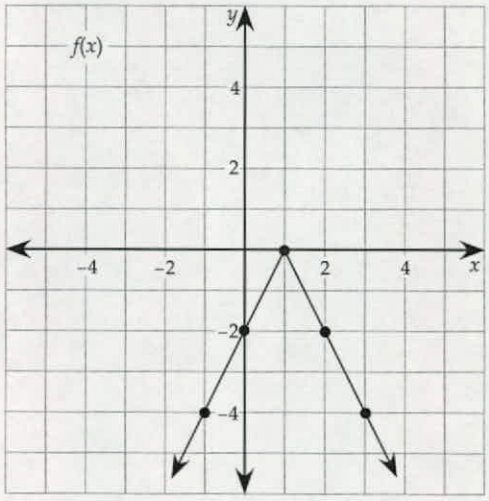


continued

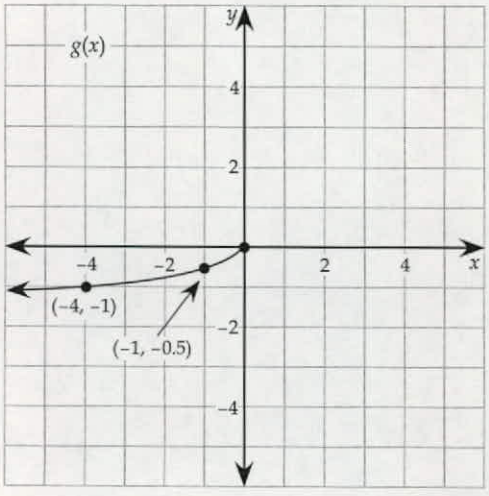
Assignment 3.1: Reflections in the x-axis and in the y-axis (continued)

5. For each of the graphs below, write the equation of the transformed function in terms of the basic function of the same type.

   a) *(3 marks)*



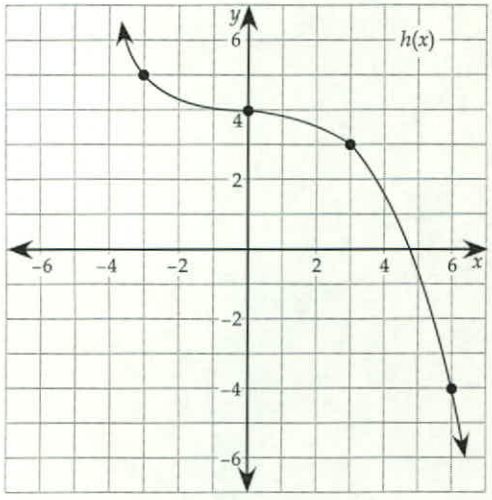
   b) *(3 marks)*



continued

Assignment 3.1: Reflections in the x-axis and in the y-axis (continued)

c) *(3 marks)*



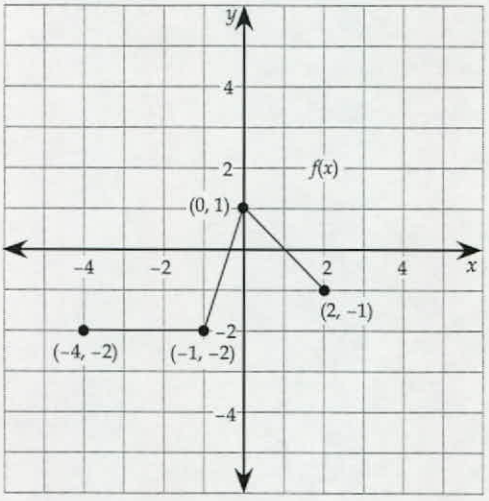
Assignment 3.2

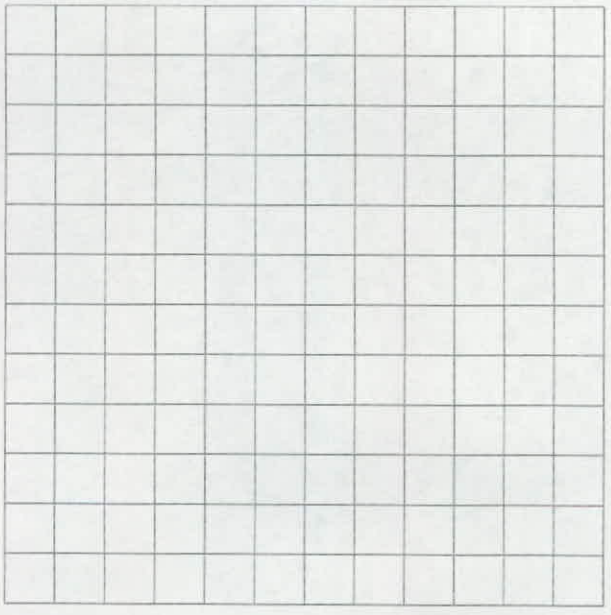
Inverse Functions and Relations

Total: 31 marks

You need to show all your steps as you work out your solutions, and make sure your answers are clear (include units, where appropriate). Check the Introduction for the Marking Guidelines that will be used for all assignments and examinations.

1. Use the graph of $f(x)$, shown below, to sketch the graph of $y = f^{-1}(x)$. *(1 mark)*



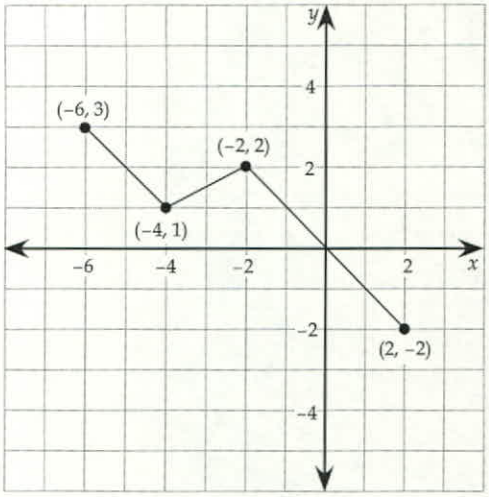


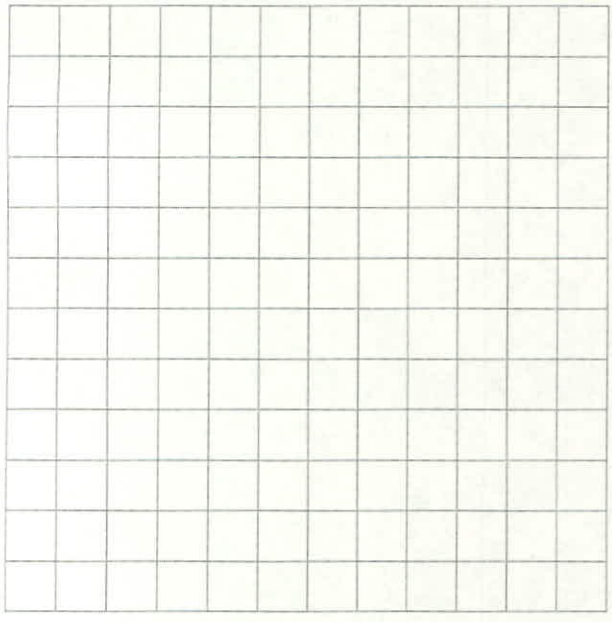
continued

Assignment 3.2: Inverse Functions and Relations (continued)

2. Reflect each of the functions below through the line $y = x$.

   a) *(1 mark)*

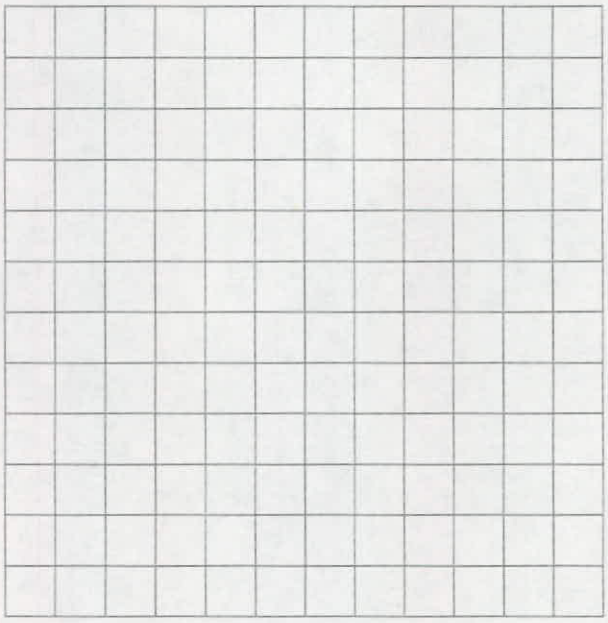


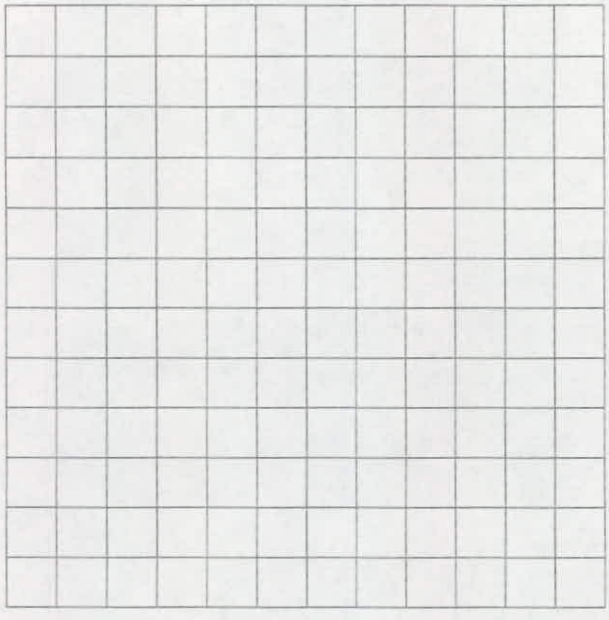


continued

Assignment 3.2: Inverse Functions and Relations (continued)

b) $g(x) = 2\sqrt{(x+1)} - 3$ *(2 marks)*





continued

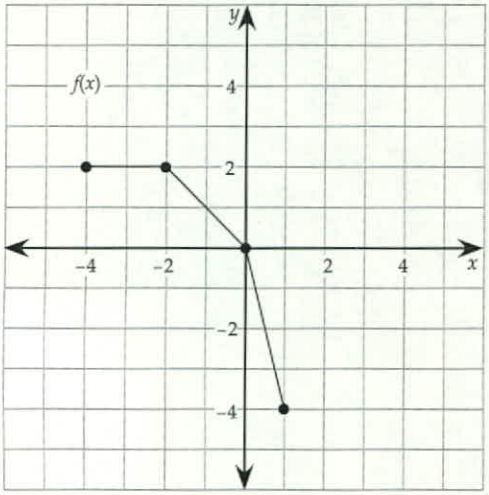
Assignment 3.2: Inverse Functions and Relations (continued)

3. Write the coordinates of the inverse of the following function. *(1 mark)*

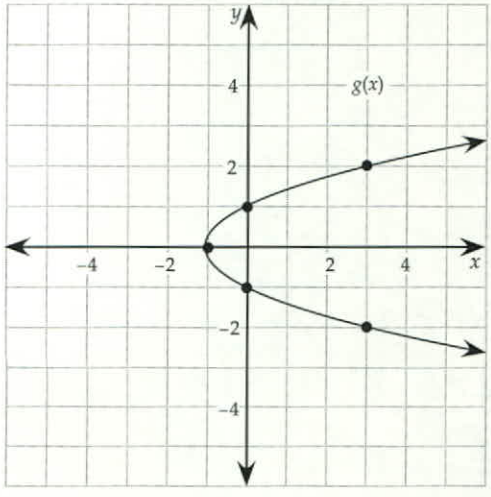
$$f(x) = \{(-3, -4), (-2, 5), (0, 0), (2, -5), (5, 7), (6, -1)\}$$

4. For each of the following relations, determine if they are one-to-one functions. *(3 × 1 mark each = 3 marks)*

a)



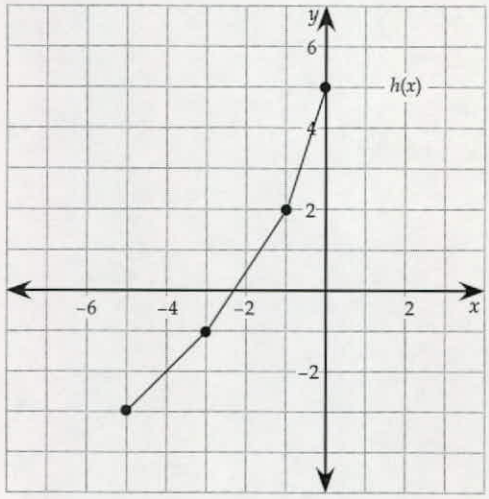
b)



continued

Assignment 3.2: Inverse Functions and Relations (continued)

c)



continued

Assignment 3.2: Inverse Functions and Relations (continued)

5. Show algebraically that the functions f and g are inverses of each other. *(2 × 2 marks each = 4 marks)*

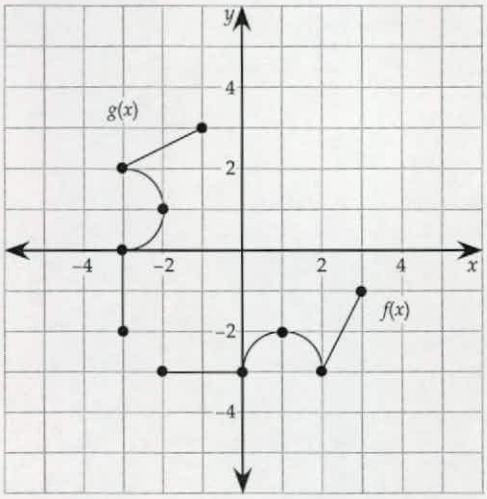
a) $f(x) = \frac{1}{2}x + 1$ $\qquad$ $g(x) = 2(x - 1)$

b) $f(x) = \frac{1}{3}(x - 1)^3$ $\qquad$ $g(x) = \sqrt[3]{3x} + 1$

continued

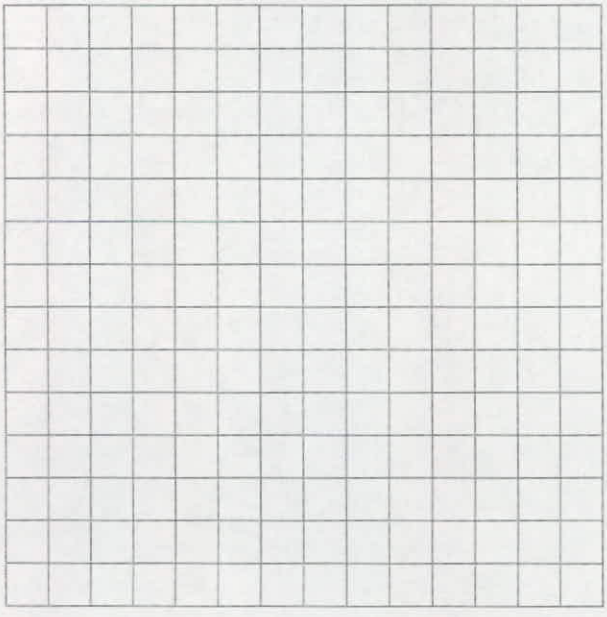
Assignment 3.2: Inverse Functions and Relations (continued)

6. Determine graphically if the following sets of relations are inverses of each other. *(1 mark)*



7. Find $f^{-1}(x)$ algebraically. Graph $f^{-1}(x)$. If necessary, write restrictions on the domain of $f(x)$ to ensure $f^{-1}(x)$ is a function.

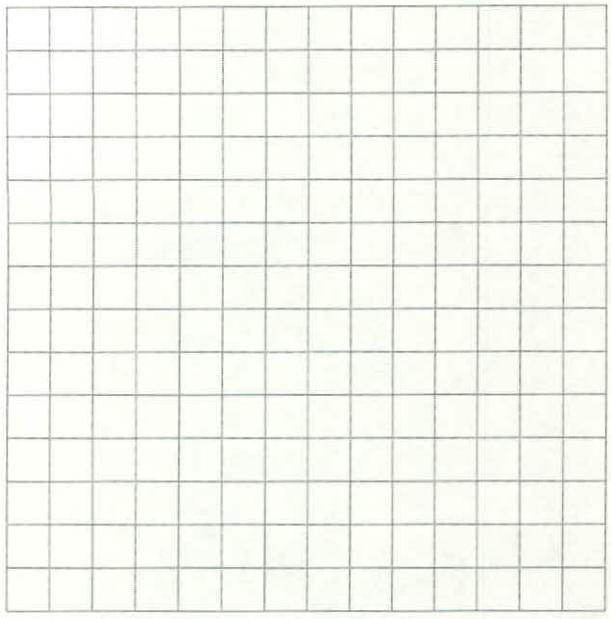
   a) $f(x) = 3x + 5$ *(2 marks)*



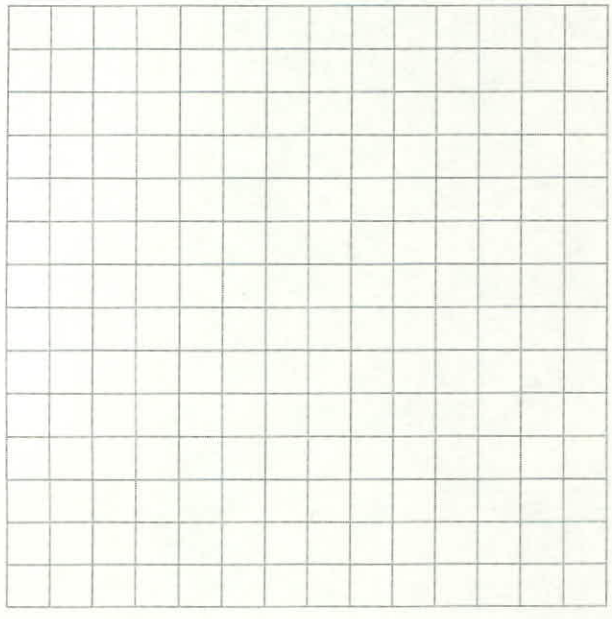
continued

Assignment 3.2: Inverse Functions and Relations (continued)

b) $f(x) = \frac{1}{2}x - 4$ *(2 marks)*



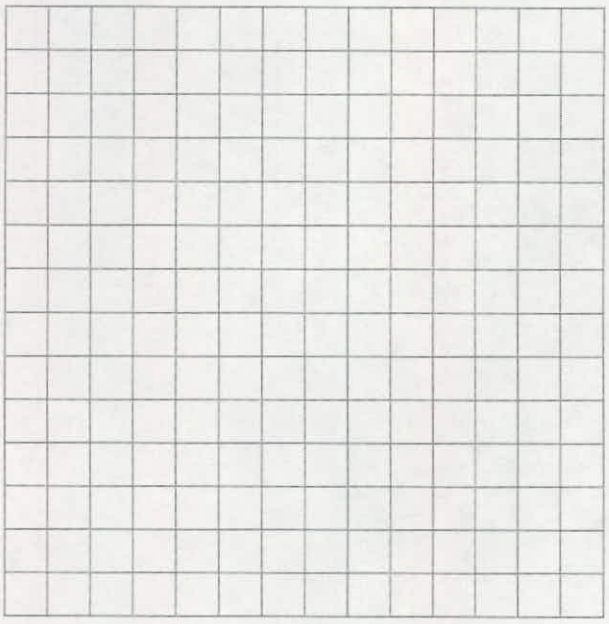
c) $f(x) = 2x^2 - 6$ *(3 marks)*



continued

Assignment 3.2: Inverse Functions and Relations (continued)

d) $f(x) = x^2 - 8x + 16$ *(3 marks)*

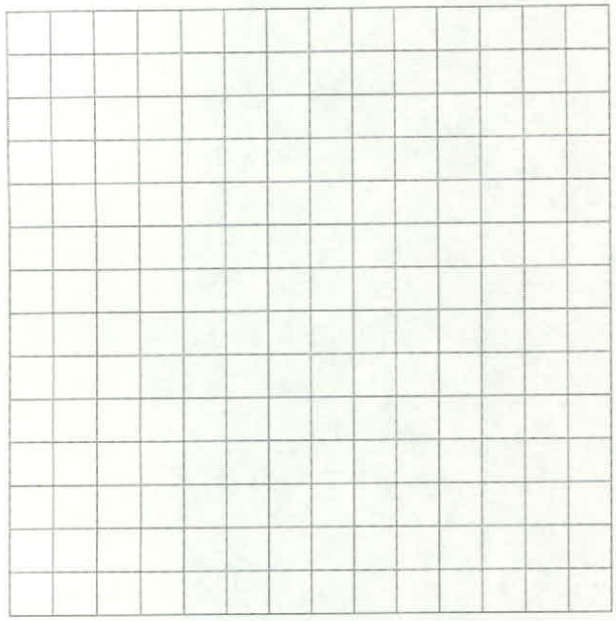


continued

Assignment 3.2: Inverse Functions and Relations (continued)

8. Find the inverses of the following functions and sketch the graph

a) $f(x) = \sqrt{2(x-1)} + 5$ *(3 marks)*



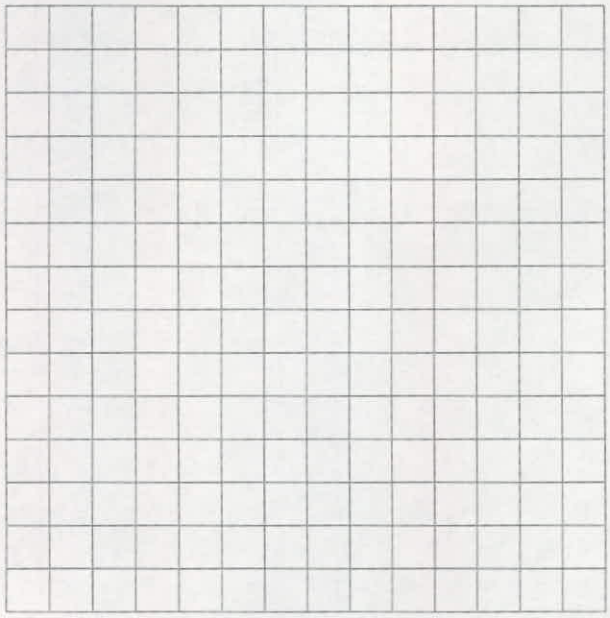
b) Explain the connection between the domain and range of $f(x)$ and $f^{-1}(x)$. *(1 mark)*

continued

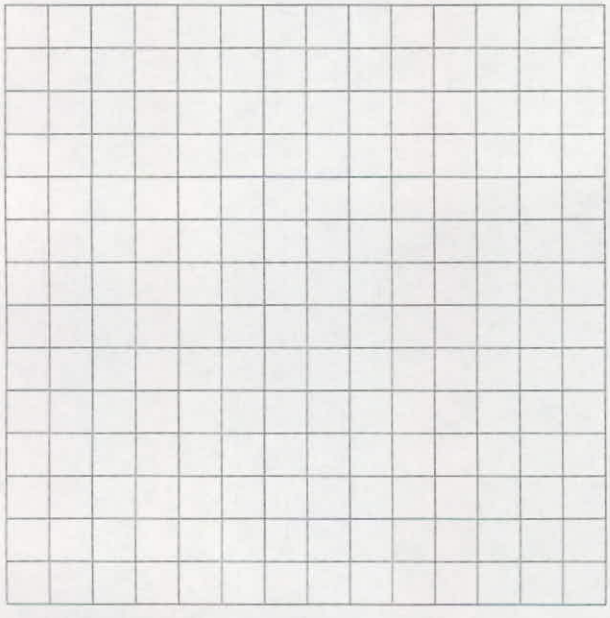
Assignment 3.2: Inverse Functions and Relations (continued)

9. Given: $g(x) = (x - 4)^2 - 2$

   a) Graph the function $g(x)$. *(1 mark)*



   b) Graph the inverse of $g(x)$. *(1 mark)*



continued

c) Is the inverse of $g(x)$ a function? Explain. *(1 mark)*

d) Determine restrictions on the domain of $g(x)$ in order for its inverse $g^{-1}(x)$ to be a function. *(1 mark)*

Module 3 Summary

In this module, you learned all about reflections. You learned about reflections through the x-axis, the y-axis, and the line $y = x$. A reflection is a different type of transformation and thus you learned about the effect various reflections have on the function $y = f(x)$.

You also learned how to find inverse functions, which are related to reflections through the line $y = x$. You were able to find inverse functions both algebraically and graphically. However, sometimes inverse relations are not functions. If a function is one-to-one, its inverse is a function. If not, you can often restrict the domain of the function to ensure its inverse is a function.

In the next module, you will be learning about polynomial functions. Linear, quadratic, and cubic functions are examples of polynomial functions. You will find patterns in the next module that extend to polynomial functions of any degree.

Submitting Your Assignments

It is now time for you to submit Assignments 3.1 and 3.2 to the Distance Learning Unit so that you can receive some feedback on how you are doing in this course. Remember that you must submit all the assignments in this course before you can receive your credit.

Make sure you have completed all parts of your Module 3 assignments and organize your material in the following order:

- ❑ Module 3 Cover Sheet (found at the end of the course Introduction)
- ❑ Assignment 3.1: Reflections in the x-axis and in the y-axis
- ❑ Assignment 3.2: Inverse Functions and Relations

For instructions on submitting your assignments, refer to How to Submit Assignments in the course Introduction.

Notes

Assignment 4.1

Polynomial Functions

Total: 40 marks

You need to show all your steps as you work out your solutions, and make sure your answers are clear (include units, where appropriate). Check the Introduction for the Marking Guidelines that will be used for all assignments and examinations.

1. Identify the polynomial functions in the following set of functions. Justify your answers. *(2 × 1 mark each = 2 marks)*

   a) $y = x^3 + 2x - \sqrt{3}$

   This is a polynomial function because the exponents are whole numbers and can be rewritten as $y = x^3 + 2x - 3$

   b) $y = \frac{1}{x^3} - 4$

   This is not a polynomial function since variable can not be left in the denominator So when the x^3 is brought to the numerator, it becomes x^{-3}. The exponent x^{-3} is not a whole number

2. State the maximum number of turns, as well as the right and left end behaviours for each of the following graphs. *(3 × 2 marks each = 6 marks)*

   a) $f(x) = -2x^3 + 3x^2 - 4x + 1$

- because the degree is odd and the leading coefficient is negative, the graph falls to the right and rises to the left just like the line $y = x$
- Because the degree is 3-1, there are 2 possible turns

continued

Assignment 4.1: Polynomial Functions (continued)

b) $g(x) = x^4 - 5x^2 + x - 7$

- Because the degree n is 4 there are 3 turns possible
- Because the degree is even and the leading coefficient is positive the graph rises to the left and right just like line $y = x^2$

c) $f(x) = x^5 - 3x^4 - x^3 - 7$

- Because the degree n is 5, there are 4 turns possible.
- Because the degree is odd and the leading coefficient is positive the graph rises to the right and falls to the left like line $y = x$

3. Match the polynomial function with the correct graph. *(6 ×0.5 mark each = 3 marks)*

a) $y = \frac{1}{4}x^3 + x^2 - \frac{7}{4}x - \frac{5}{2}$ = V

b) $y = -\frac{1}{4}x^3 + x^2 - \frac{7}{4}x - \frac{5}{2}$ = iii

c) $y = x^4 - 3x^3 + 2$ = ii

d) $y = -x^4 + 4x^3 - 3x^2 + 1$ = iv

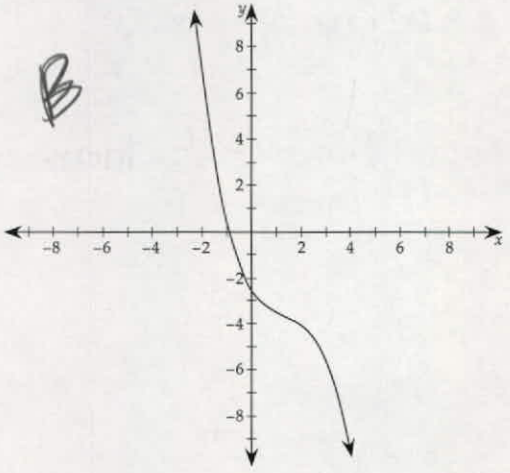
e) $y = x^5 - 2x^4 - 3x^3 + 5x^2 + 4x - 1$ = vi

f) $y = -x^5$ = i

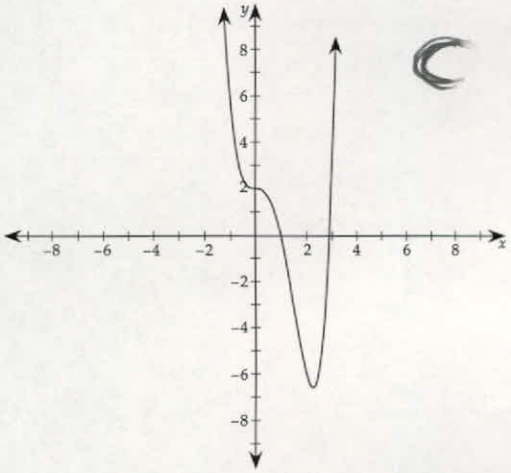
continued

Assignment 4.1: Polynomial Functions (continued)

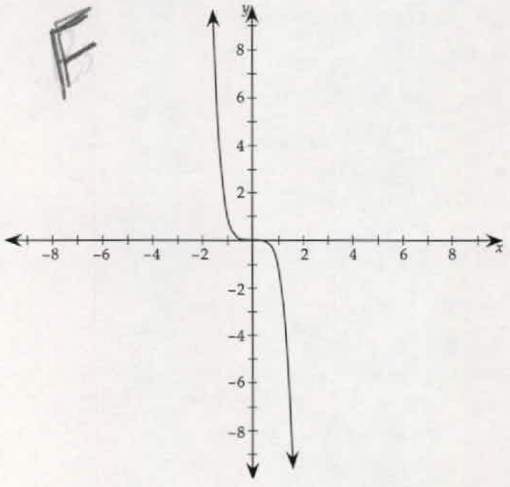
i) B



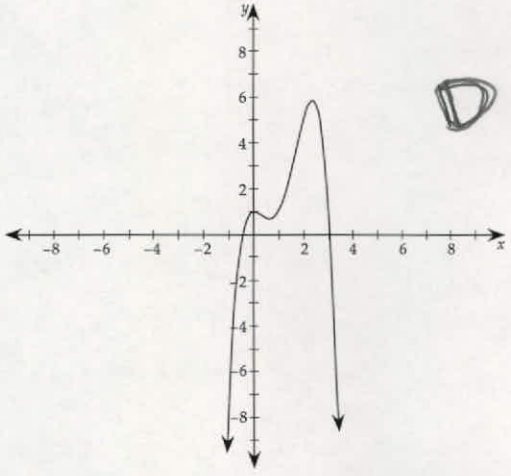
ii) C



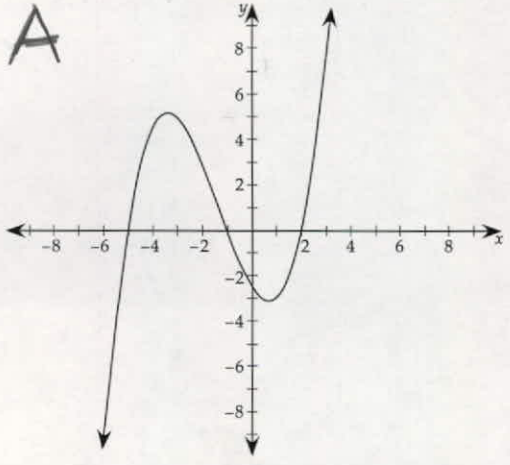
iii) F



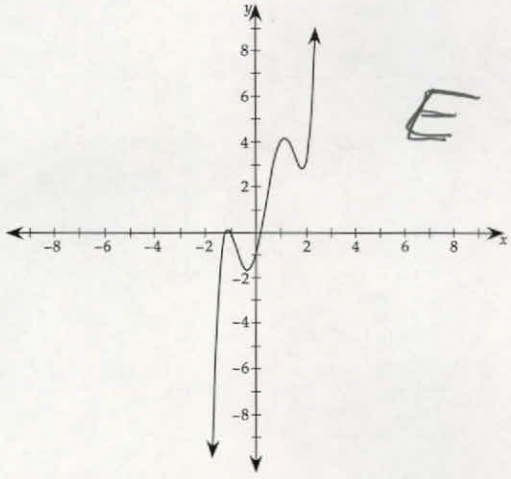
iv) D



v) A



vi) E

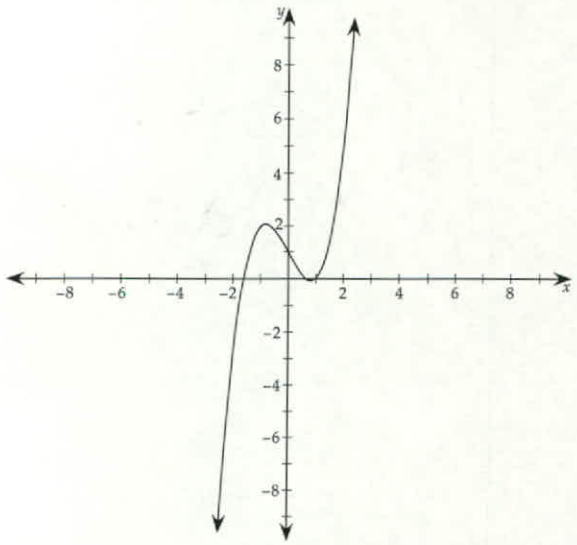


continued

Assignment 4.1: Polynomial Functions (continued)

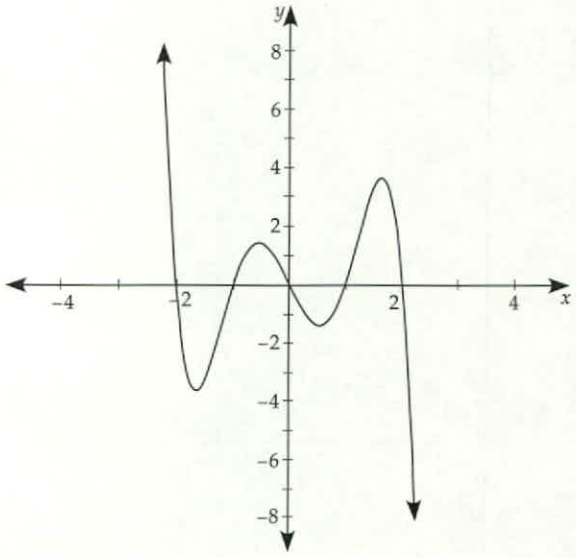
4. Analyze the following graphs to determine the degree and the sign of the leading coefficient of each function. *(6 × 1 mark each = 6 marks)*

a)



- odd degree
- positive coefficient

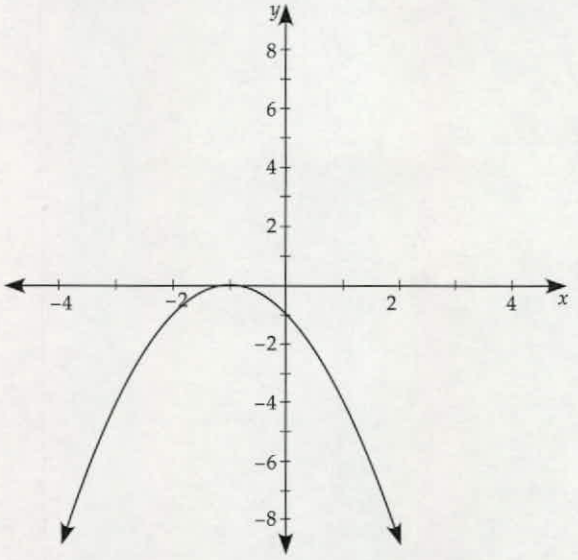
b)



- negative coefficient
- odd degree

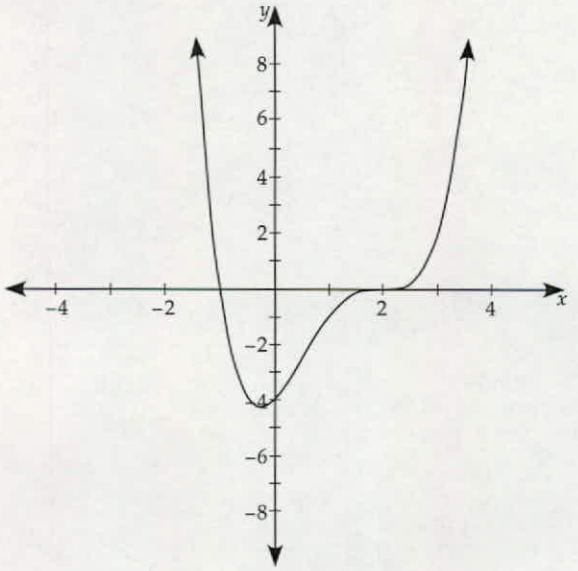
continued

c)



- degree is even
- leading coefficient is negative

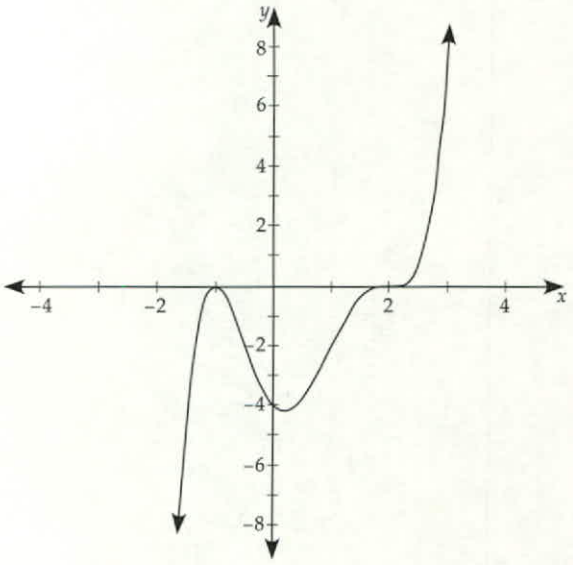
d)



- degree is even
- leading coefficient is positive

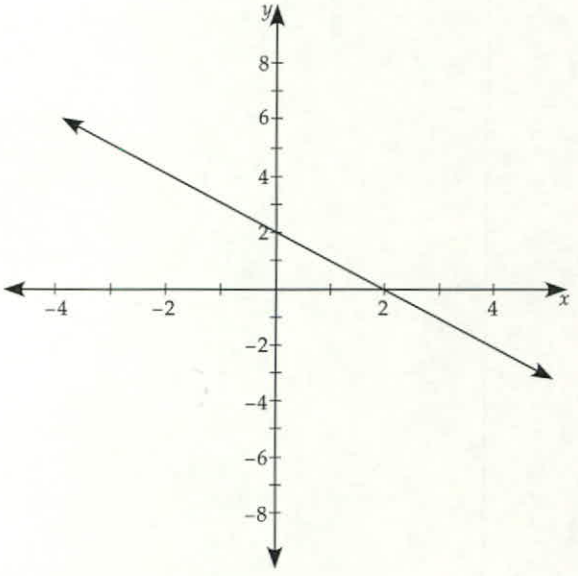
continued

e)



- odd degree
- positive coefficient

f)



- negative coefficient
- odd degree

continued

Assignment 4.1: Polynomial Functions (continued)

5. Divide using long division and write your answer in the form dividend = (divisor)(quotient) + remainder.

a) $(4x^3 - 2x^2 + 3x - 1) \div (x - 1)$ *(3 marks)*

$$
\begin{array}{r}
4x^2+2x+5 \\
x-1 \overline{)\,4x^3-2x^2+3x-1} \\
-4x^3+4x^2 \\
\hline
2x^2+3x-1 \\
-2x^2+2x \\
\hline
5x-1 \\
-5x+5 \\
\hline
4
\end{array}
$$

$4x^2+2x+5$ with remainder of 4

b) $(8x^4 - 2x^2 + 2x + 4) \div (2x - 3)$ *(4 marks)*

$$
\begin{array}{r}
4x^3+6x^2+8x+13 \\
2x-3 \overline{)\,8x^4+0x^3-2x^2+2x+4} \\
-8x^4+12x^3 \\
\hline
12x^3-2x^2+2x+4 \\
-12x^3+18x^2 \\
\hline
16x^2+2x+4 \\
-16x^2+24x \\
\hline
26x+4 \\
-26x+39 \\
\hline
43
\end{array}
$$

$4x^3+6x^2+8x+13$ - remainder of 43

c) $-35 - 8x - 22x^2 + 3x^4 - 2x^5 \div (x + 2)$ *(4 marks)*

continued

Assignment 4.1: Polynomial Functions (continued)

6. Use synthetic division to find the quotient and the remainder. Write your answer in the form dividend = (divisor)(quotient) + remainder. *(3 × 4 marks each = 12 marks)*

   a) $(x^3 - x^2 + 2) \div (x + 1)$

   b) $(2x^4 + 4x^3 - x^2 - 49) \div (x - 2)$

   c) $(-x^4 - 15x + 20) \div (x + 3)$

Assignment 4.2

Factoring and Graphing Polynomials

Total: 42 marks

You need to show all your steps as you work out your solutions, and make sure your answers are clear (include units, where appropriate). Check the Introduction for the Marking Guidelines that will be used for all assignments and examinations.

1. Use the Remainder Theorem to determine the remainder when each of the following polynomials are divided by $x - 6$. *(2 × 1 mark each = 2 marks)*

   a) $f(x) = x^3 - 2x^2 - 21x - 18$

   $x = 6$

   $= (6)^3 - 2(6)^2 - 21(6) - 18$

   $= 216 - 2 \times 36 - 126 - 18$

   $= 216 - 72 - 126 - 18$

   $= 0$

   remainder is 0

   b) $g(x) = x^4 - 8x^3 + 5x^2 + 18x + 114$

   $= (6)^4 - 8(6)^3 + 5(6)^2 + 18(6) + 114$

   $= 1296 - 8 \times 6^3 + 180 + 108 + 114$

   $= 1698 - 8 \times 216$

   $= 1698 - 1728 = -30$

2. Use the Factor Theorem to determine whether $x + 4$ is a factor of each of the following polynomials. *(2 × 2 marks each = 4 marks)*

   a) $f(x) = x^3 + 10x^2 + 32x + 32$

   $= (-4)^3 + 10(-4)^2 + 32(-4) + 32$

   $= -64 + 160 - 128 + 32 = 0$

   Yes

   b) $g(x) = x^5 + 7x^4 - 17x^3 - 119x^2 + 16x + 108$

   $(-4)^5 + 7(-4)^4 - 17(-4)^3 - 119(-4)^2 + 16(-4) + 108$

   $= -4$

   NO

continued

Assignment 4.2: Factoring and Graphing Polynomials (continued)

3. Factor completely:

   a) $f(x) = x^5 + 5x^4 - 21x^3 - 137x^2 - 88x + 240$, knowing that $f(5) = 0$ and $f(-3) = 0$.
   (5 marks)

continued

Assignment 4.2: Factoring and Graphing Polynomials (continued)

b) $f(x) = x^3 + 8x^2 + 5x - 50$ *(3 marks)*

continued

Assignment 4.2: Factoring and Graphing Polynomials (continued)

c) $f(x) = -x^4 - 4x^3 + 19x^2 + 46x - 120$, knowing that $(x + 5)$ and $(x - 2)$ are factors.
(5 marks)

continued

Assignment 4.2: Factoring and Graphing Polynomials (continued)

4. a) For the function $f(x) = x^3 - 9x^2 + 23x - 15$, find:

i) the zeros *(3 marks)*

$x = 1$

$x = 3$

$x = 5$

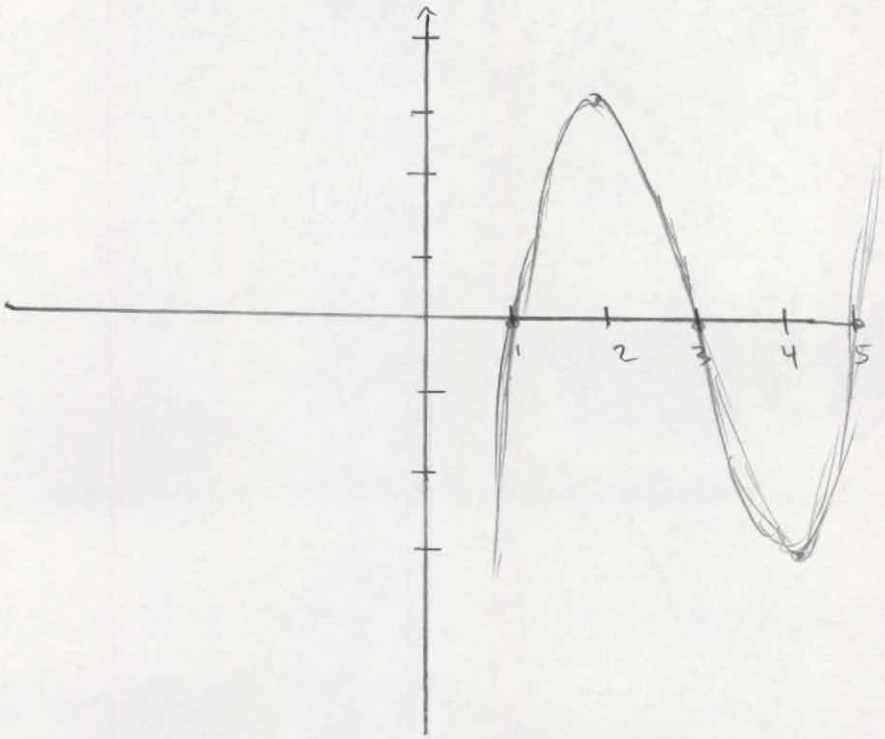
ii) the y-intercept *(1 mark)*

-15

iii) left-right end behaviour *(1 mark)*

left falls down
right rises up

iv) the sketch of the graph *(2 marks)*



continued

Assignment 4.2: Factoring and Graphing Polynomials (continued)

b) For the function $f(x) = (x + 3)^2(2x + 1)(x - 1)$, find:

i) the zeros *(2 marks)*

$x = -3$

$x = -\frac{1}{2}$

$x = 1$

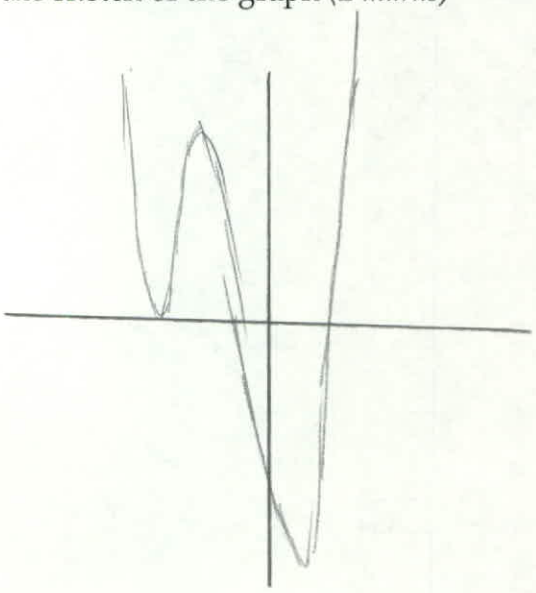
ii) the y-intercept *(1 mark)*

$= -9$

iii) left-right end behaviour *(1 mark)*

both sides rise

iv) the sketch of the graph *(2 marks)*



continued

Assignment 4.2: Factoring and Graphing Polynomials (continued)

c) For the function $f(x) = -(x^2 - 6x + 9)(x^2 - x - 6)$, find:

i) the zeros *(2 marks)*

$x = 3$

$x = -2$

$x = 3$

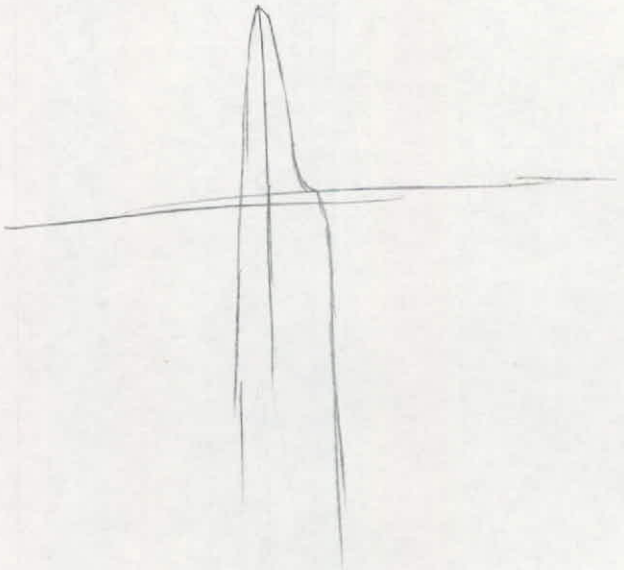
ii) the y-intercept *(1 mark)*

54

iii) left-right end behaviour *(1 mark)*

both sides fall down into the negative

iv) the sketch of the graph *(2 marks)*



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Assignment 4.2: Factoring and Graphing Polynomials (continued)

5. Three consecutive integers multiply together to give −120.
   a) Determine a polynomial equation to represent this situation. *(1 mark)*

   b) Using your equation from part (a), determine the three consecutive integers that multiply together to give −120. *(3 marks)*

Module 4 Summary

In this module, you learned more about cubic, quartic, and quintic polynomial functions. You learned about the characteristics of each of these functions, as well as techniques you can use to help graph these functions. In order to determine the zeros of each of these functions, you needed to first learn how to divide polynomials by a binomial using long division or synthetic division. Then, using the factor theorem and the remainder theorem, you were able to completely factor polynomial functions.

After you write your midterm examination, you will start the second half of the course. In Module 5, you will be learning about trigonometric functions and the unit circle.

Submitting Your Assignments

It is now time for you to submit Assignments 4.1 and 4.2 to the Distance Learning Unit so that you can receive some feedback on how you are doing in this course. Remember that you must submit all the assignments in this course before you can receive your credit.

Make sure you have completed all parts of your Module 4 assignments and organize your material in the following order:

- ❑ Module 4 Cover Sheet (found at the end of the course Introduction)
- ❑ Assignment 4.1: Polynomial Functions
- ❑ Assignment 4.2: Factoring and Graphing Polynomials

For instructions on submitting your assignments, refer to How to Submit Assignments in the course Introduction.

Midterm Examination

Congratulations, you have finished Module 4 in the course. The midterm examination is out of 100 marks and worth 20% of your final mark. In order to do well on this examination, you should review all of your learning activities and assignments from Modules 1 to 4.

You will complete this examination while being supervised by a proctor. You should already have made arrangements to have the examination sent to the proctor from the Distance Learning Unit. If you have not yet made arrangements to write it, then do so now. The instructions for doing so are provided in the Introduction to this module.

You will need to bring the following items to the examination: pens/pencils (2 or 3 of each), blank paper, a ruler, a scientific calculator, and your Examination Resource Sheet. A maximum of 3 hours is available to complete your midterm examination. When you have completed it, the proctor will then forward it for assessment. Good luck!

At this point you will also have to combine your resource sheets from Modules 1 to 4 onto one 8½" × 11" paper (you may use both sides). Be sure you have all the formulas, definitions, and strategies that you think you will need. This paper can be brought into the examination with you. We suggest that you divide your paper into two quadrants on each side so that each quadrant contains information from one module.

Examination Review

You are now ready to begin preparing for your midterm examination. Please review the content, learning activities, and assignments from Modules 1 to 4.

The midterm practice examination is also an excellent study aid for reviewing Modules 1 to 4.

You will learn what types of questions will appear on the examination and what material will be assessed. Remember, your mark on the midterm examination determines 20% of your final mark in this course and you will have 3 hours to complete the examination.

Midterm Practice Examination and Answer Key

To help you succeed in your examination, a practice examination can be found in the learning management system (LMS). The midterm practice examination is very similar to the actual examination that you will be writing. The answer key is also included so that, when you have finished writing the practice examination, you can check your answers. This will give you the confidence that you need to do well on your examination. If you do not have access to the Internet, contact the Distance Learning Unit at 1-800-465-9915 to get a copy of the practice examination and the answer key.

To get the most out of your midterm practice examination, follow these steps:

1. Study for the midterm practice examination as if it were an actual examination.
2. Review those learning activities and assignments from Modules 1 to 4 that you found the most challenging. Reread those lessons carefully and learn the concepts.
3. Contact your learning partner and your tutor/marker if you need help.
4. Review your lessons from Modules 1 to 4, including all of your notes, learning activities, and assignments.
5. Use your module resource sheets to make a draft of your Midterm Examination Resource Sheet. You can use both sides of an 8½" by 11" piece of paper.
6. Bring the following to the midterm practice examination: pens/pencils (2 or 3 of each), blank paper, a ruler, a scientific calculator, and your Midterm Examination Resource Sheet.
7. Write your midterm practice examination as if it were an actual examination. In other words, write the entire examination in one sitting, and don't check your answers until you have completed the entire examination. Remember that the time allowed for writing the midterm examination is 3 hours.
8. Once you have completed the entire practice examination, check your answers against the answer key. Review the questions that you got wrong. For each of those questions, you will need to go back into the course and learn the things that you have missed.
9. Go over your resource sheet. Was anything missing or is there anything that you didn't need to have on it? Make adjustments to your Midterm Examination Resource Sheet. Once you are happy with it, make a photocopy that you can keep.

Notes

Assignment 5.1

Degrees, Radians, and the Unit Circle

Total: 31 marks

You need to show all your steps as you work out your solutions, and make sure your answers are clear (include units, where appropriate). Check the Introduction for the Marking Guidelines that will be used for all assignments and examinations.

1. Convert each of the following angles to radians. Simplify and write the exact answer. *(2 × 1 mark each = 2 marks)*

   a) $-572°$ $= -\frac{143\pi}{45} = -9.98328$

   b) $724°$ $724° \times \frac{\pi}{180°} = \frac{181\pi}{45} = 12.63618$

2. Convert each of the following angles to degrees. *(2 × 1 mark each = 2 marks)*

   a) $-\frac{4\pi}{9}$ $= -\frac{4\pi}{9} \times \frac{180°}{\pi}$

   $= -4 \times 20° = -80°$

   b) $\frac{7\pi}{3}$ $= \frac{7\pi}{3} \times \frac{180°}{\pi} = 7 \times 60° = 420°$

3. Explain how you would draw an angle of 3 radians on any given circle. *(1 mark)*

continued

Assignment 5.1: Degrees, Radians, and the Unit Circle (continued)

4. Determine all of the angles that are coterminal with the given angle over the domain $[-720°, 720°]$. *(2 × 3 marks each = 6 marks)*

   a) 310°

$310° - 360° = -50°$

$-50° - 360° = -410°$

$310° + 360° = 670°$

   b) −681°

$-681° + 360° = -321°$

$-321° + 360° = 39°$

$39° + 360° = 399°$

5. Determine all of the angles that are coterminal with the given angle over the domain $[-4\pi, 4\pi]$. *(2 × 3 marks each = 6 marks)*

   a) $-\frac{\pi}{6}$

continued

b) $\frac{3\pi}{4}$

6. Express the angles that are coterminal with the following angles in general form. *(2 × 1 mark each = 2 marks)*

a) $\frac{11\pi}{3}$

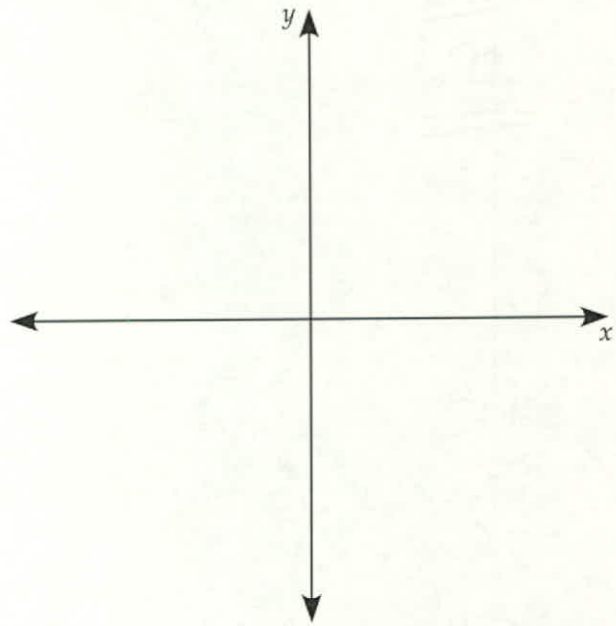
b) 276°

continued

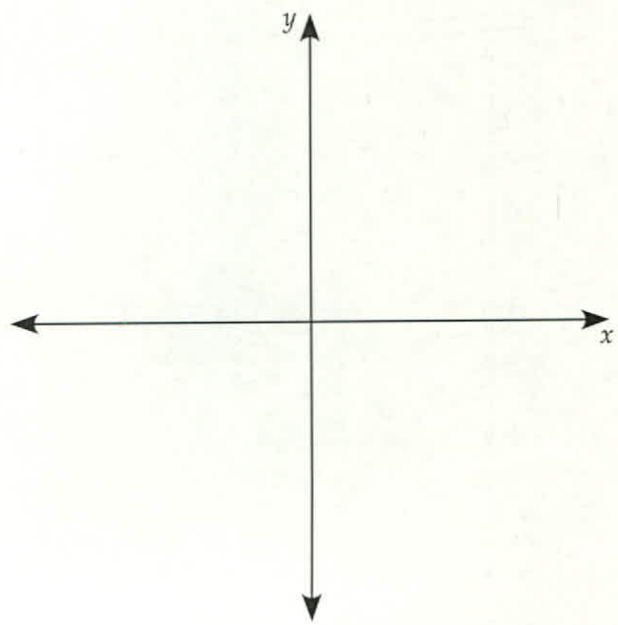
Assignment 5.1: Degrees, Radians, and the Unit Circle (continued)

7. Sketch the following angles. Be sure to state the coterminal angle. Determine the exact coordinates on the unit circle corresponding to each of the following angles. *(4 × 2 marks each = 8 marks)*

   a) 600°



   b) $-\frac{17\pi}{6}$



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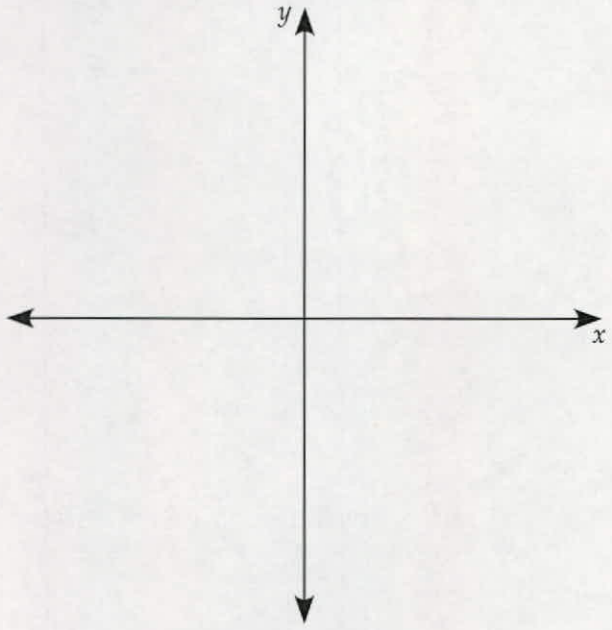
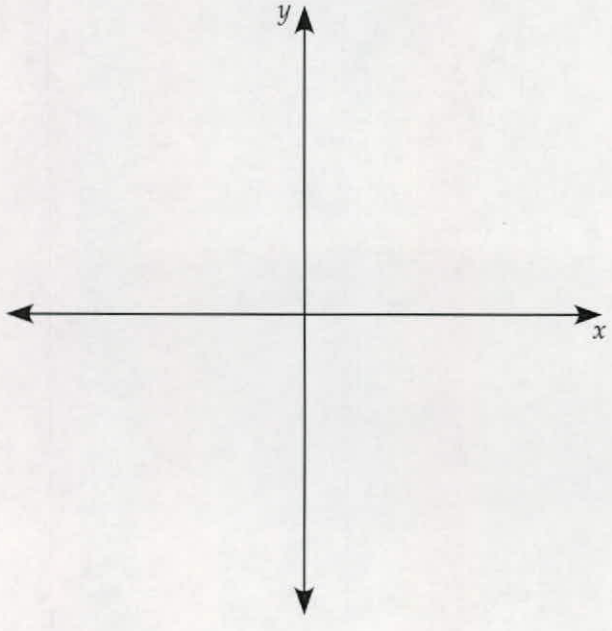
c) 12π



d) $-225°$



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Assignment 5.1: Degrees, Radians, and the Unit Circle (continued)

8. Show that the point with coordinates $\left(\frac{\sqrt{11}}{6}, \frac{5}{6}\right)$ is on the unit circle. *(2 marks)*

9. Is the point $\left(\frac{5}{4}, \frac{1}{4}\right)$ on the unit circle? Explain without doing any calculations.

   (1 mark)

10. Determine the equation of a circle centred at the origin with a radius of 5 cm.
    (1 mark)

Assignment 5.2

The Six Trigonometric Ratios

Total: 32 marks

You need to show all your steps as you work out your solutions, and make sure your answers are clear (include units, where appropriate). Check the Introduction for the Marking Guidelines that will be used for all assignments and examinations.

1. Use the coordinates of the points on the unit circle and the definitions of the circular functions to find the following values.

   a) $\sin 390°$ *(2 marks)* $= \frac{1}{2}$

   b) $\cos\left(-\frac{5\pi}{6}\right)$ *(2 marks)* $= -\frac{\sqrt{3}}{2}$

   c) $\tan 0°$ *(1 mark)* 0

   d) $\csc\frac{\pi}{3}$ *(1 mark)* $= \frac{2\sqrt{3}}{3}$

   e) $\sec 210°$ *(1 mark)* $= \frac{1}{\cos 210°} = \frac{-\sqrt{3}}{2} = \sec 210 = -\frac{2}{\sqrt{3}}$

   f) $\cot\left(-\frac{\pi}{2}\right)$ *(2 marks)* 0

continued

Assignment 5.2: The Six Trigonometric Ratios (continued)

2. Determine the *exact* value of the following expression. *(4 marks)*

$$\sin^3\left(-\frac{\pi}{2}\right) + 2\cos^2\left(\frac{\pi}{4}\right)\cot^2\left(-\frac{\pi}{4}\right)$$

$(-1)^3 + 2 \times \left(\frac{\sqrt{2}}{2}\right)^2 \times (-1)^2$

$-1 + 2 \times \frac{2}{4} \times 1$

$-1 + 2 \times \frac{1}{2}$

$-1 + 1$

$= 0$

3. Find an approximate value, rounded to 5 decimal places. *(6 × 1 mark each = 6 marks)*

a) $\sin\frac{9\pi}{13}$

0.82298

b) $\cos 582°$

−0.74315

c) $\tan 7$

0.87145

continued

Assignment 5.2: The Six Trigonometric Ratios (continued)

d) csc 51°

1.28676

e) $\sec \frac{\pi}{10}$

1.05146

f) cot 81°

0.158385

4. Find the exact values of the remaining five circular functions if given the following information. *(4 marks)*

$$\sin \theta = -\frac{5}{13} \text{ and } \tan \theta < 0$$

continued

Assignment 5.2: The Six Trigonometric Ratios (continued)

5. Solve the following equations over the indicated intervals. All answers must be *exact* special values. *(2 × 2 marks each = 4 marks)*

   a) $\sec\theta = -\frac{2}{\sqrt{2}}, 0 \le \theta < 2\pi$

   b) $\tan\theta = 0, -720° \le \theta < 0$

6. Determine the measures of all the angles of rotation in the standard position over the interval $-4\pi \le \theta < 0$ that correspond to the coordinates $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$. *(2 marks)*

7. Determine the exact values of the six trigonometric ratios, given that the point $\left(\frac{2\sqrt{6}}{5}, -\frac{1}{5}\right)$ is a point on the terminal arm of an angle in standard position. *(3 marks)*

$\sin\theta = \frac{-1}{5}$

$\cos\theta = -\frac{2\sqrt{6}}{5}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{-1/5}{\frac{2\sqrt{6}}{5}} = -\frac{1}{5} \times \frac{5}{2\sqrt{6}} = -\frac{\sqrt{6}}{12}$

$\csc\theta = \frac{1}{\sin\theta} = -\frac{12}{\sqrt{6}}$

$\sec\theta = \frac{1}{\cos\theta} = -\frac{5}{2\sqrt{6}}$

$\cot\theta = \frac{1}{\tan\theta} = \frac{12}{\sqrt{6}}$

Assignment 5.3

Trigonometric Functions and Their Graphs

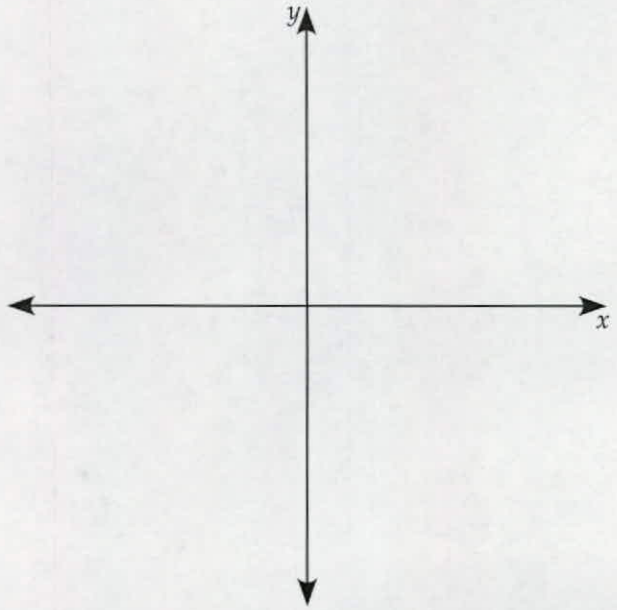
Total: 26 marks

You need to show all your steps as you work out your solutions, and make sure your answers are clear (include units, where appropriate). Check the Introduction for the Marking Guidelines that will be used for all assignments and examinations.

1. Fill in the property chart for each function. Sketch the following curves.

   a) $y = -2 \sin\left(\frac{\pi}{2}x\right) + 4$ *(6 marks)*

	$y = -2 \sin\left(\frac{\pi}{2}x\right) + 4$
Period	
Amplitude	
Phase Shift (Horizontal)	
Domain	
Range	
y-intercept	

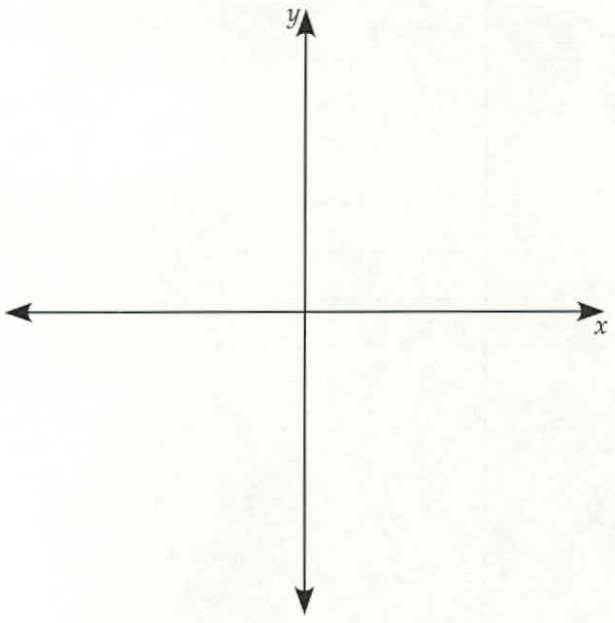


continued

Assignment 5.3: Trigonometric Functions and Their Graphs (continued)

b) $y = \cos\left(\frac{1}{2}(x - \pi)\right) - 3$ *(6 marks)*

	$y = \cos\left(\frac{1}{2}(x - \pi)\right) - 3$
Period	
Amplitude	
Phase Shift (Horizontal)	
Domain	
Range	
y-intercept	



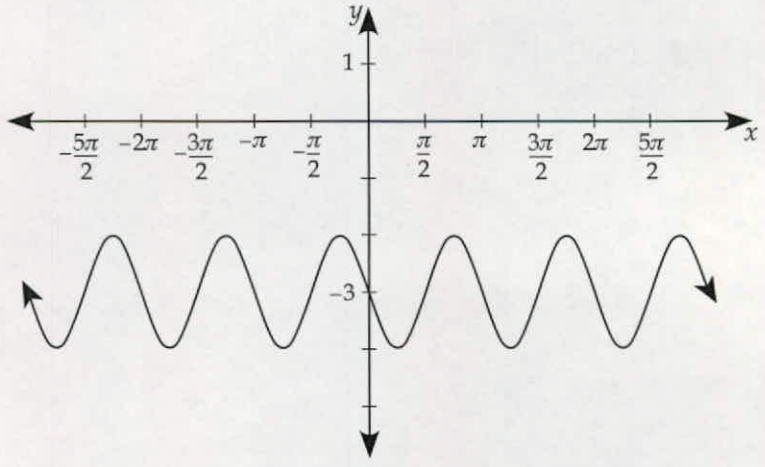
continued

2. Find the amplitude, period, horizontal shift, and vertical shift for the following functions. *(2 × 2 marks each = 4 marks)*

   a) $y = -5 \sin\left(\frac{\pi}{4}\left(x + \frac{3\pi}{2}\right)\right) - 2$

   b) $y = 2 \cos\left(-2\left(x - \frac{\pi}{6}\right)\right) + 3$

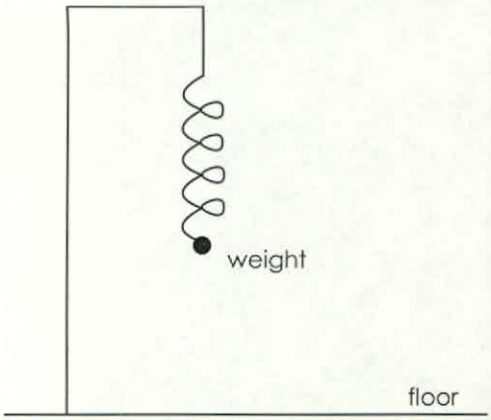
3. Find an equation of the following graph as a sine function, and then as a cosine function. *(4 marks)*



continued

Assignment 5.3: Trigonometric Functions and Their Graphs (continued)

4. A weight attached to the end of a spring is bouncing up and down. As it bounces, the weight's distance from the floor varies sinusoidally. You begin measuring the weight's distance from the floor when it is at its lowest point, 20 cm above the floor. One second after you start timing, the weight reaches its highest point, 50 cm above the floor. You could model the height as a function of time.

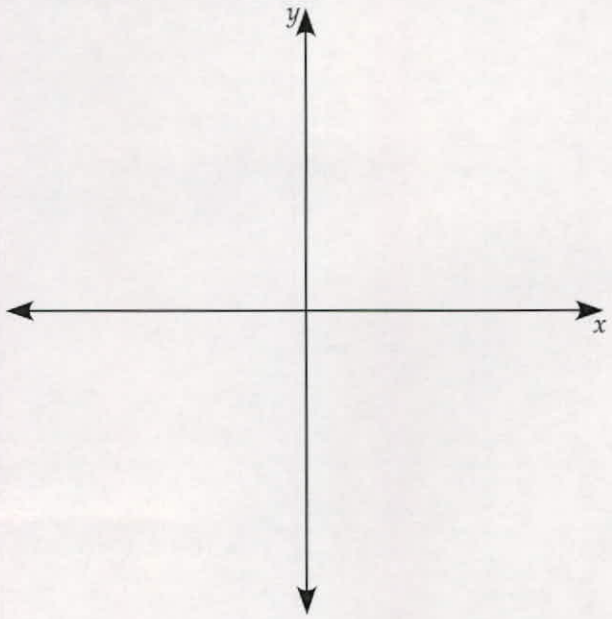


a) What is the amplitude of this function? *(1 mark)*

b) What is the period of this function? *(1 mark)*

continued

c) Sketch at least one period of the graph of this function. *(1 mark)*



d) Write a possible formula for the distance of the weight above the ground t seconds after you began your measurements. *(2 marks)*

e) What will be the weight's distance above the floor 1.5 seconds after you began your measurements? *(1 mark)*

Notes

Module 5 Summary

Congratulations, you have finished the first module in the second half of this course!

In this module, you learned how the six trigonometric ratios—sine, cosine, tangent, cosecant, secant, and cotangent—were related to the unit circle. You then studied these ratios as functions. With the sine and cosine functions, you performed transformations and analyzed the properties of each function.

In the next module, you will continue to study the topic of trigonometry with a focus on trigonometric identities built from the unit circle.

Submitting Your Assignments

It is now time for you to submit Assignments 5.1 to 5.3 to the Distance Learning Unit so that you can receive some feedback on how you are doing in this course. Remember that you must submit all the assignments in this course before you can receive your credit.

Make sure you have completed all parts of your Module 5 assignments and organize your material in the following order:

- ❑ Module 5 Cover Sheet (found at the end of the course Introduction)
- ❑ Assignment 5.1: Degrees, Radians, and the Unit Circle
- ❑ Assignment 5.2: The Six Trigonometric Ratios
- ❑ Assignment 5.3: Trigonometric Functions and Their Graphs

For instructions on submitting your assignments, refer to How to Submit Assignments in the course Introduction.

Notes

Assignment 6.1

Solving Trigonometric Equations

Total: 21 marks

You need to show all your steps as you work out your solutions, and make sure your answers are clear (include units, where appropriate). Check the Introduction for the Marking Guidelines that will be used for all assignments and examinations.

1. Solve the following equations over the indicated intervals. Provide exact answers wherever possible. Round to 3 decimal places when necessary.

   a) $\tan x = \sqrt{3}, \quad 0 \leq x < 2\pi$ *(1 mark)*

   $x = \frac{\pi}{3}$

   b) $\sec x = 2.5, \quad 0° \leq x < 180°$ *(2 marks)*

   c) $\csc x = \sec x, \quad -\pi \leq \theta < \pi$ *(2 marks)*

continued

Assignment 6.1: Solving Trigonometric Equations (continued)

2. Find the general solution to each of the following equations.

   a) $\sin\theta = -\frac{1}{\sqrt{2}}$ *(2 marks)*

   b) $\cos^2\theta - \cos\theta = 0$ *(3 marks)*

3. Solve the following equations over the indicated intervals. Provide exact answers.

   a) $2\sin^2\theta - \sin\theta = 1, \quad -360° \leq \theta < 0°$ *(4 marks)*

   b) $\cos x \tan x + \cos x = 0, \quad 0 \leq x < 2\pi$ *(3 marks)*

continued

Assignment 6.1: Solving Trigonometric Equations (continued)

4. Consider the function $y = -\cos\left(x + \frac{3\pi}{2}\right) - 1$.

   a) Determine the x-intercepts of the above function by graphing. *(3 marks)*

   b) Explain how you can determine the general solution of the equation $\cos\left(x + \frac{3\pi}{2}\right) = -1$, using the information you found in (a). *(1 mark)*

Notes

Assignment 6.2

Using Elementary Identities

Total: 44 marks

You need to show all your steps as you work out your solutions, and make sure your answers are clear (include units, where appropriate). Check the Introduction for the Marking Guidelines that will be used for all assignments and examinations.

1. Use the method of Pythagorean identities to find the exact values of the other remaining circular functions, given that $\sec\theta = \frac{13}{7}$ and $\cot\theta < 0$. *(4 marks)*

continued

Assignment 6.2: Using Elementary Identities (continued)

2. Prove that each of the following is an identity.

   a) $\sin x \tan x \csc x \cot x = 1$ *(2 marks)*

   b) $\sin^2 x \sec^2 x + \sin^2 x \csc^2 x = \sec^2 x$ *(2 marks)*

continued

Assignment 6.2: Using Elementary Identities (continued)

c) $\cot x + \tan x = \sec x \csc x$ *(2 marks)*

d) $\dfrac{1}{\cos x} - \cos x = \tan x \sin x$ *(2 marks)*

continued

Assignment 6.2: Using Elementary Identities (continued)

e) $\csc x = \dfrac{1 + \sec x}{\sin x + \tan x}$ *(3 marks)*

f) $\dfrac{1}{\cot x - \csc x} + \dfrac{1}{\cot x + \csc x} = -2 \cot x$ *(4 marks)*

continued

Assignment 6.2: Using Elementary Identities (continued)

3. Simplify each expression as much as possible. Determine the non-permissible values of each expression.

a) $\dfrac{\csc^2 x - 1}{\csc^2 x}$ *(4 marks)*

b) $\dfrac{\sin x \sec x}{\tan x + \cot x}$ *(5 marks)*

continued

Assignment 6.2: Using Elementary Identities (continued)

4. Consider the equation $\frac{\cos x}{1-\sin x} = \frac{1+\sin x}{\cos x}$.

   a) Verify this equation holds for $x = \frac{\pi}{6}$. *(3 marks)*

   b) Verify this equation is a trigonometric identity, and thus holds for all values of x. *(3 marks)*

continued

Assignment 6.2: Using Elementary Identities (continued)

5. Solve each equation over the indicated interval. Use exact values where possible; otherwise, round to two decimal places.

   a) $\csc^2 \theta = 2 \cot \theta, 0° \leq \theta < 360°$ *(4 marks)*

continued

Assignment 6.2: Using Elementary Identities (continued)

b) $1 - \tan\theta = \sqrt{2}\sec\theta, 0 \leq \theta < 2\pi$ (Make sure you check your solutions for extraneous roots). *(6 marks)*

Assignment 6.3

Sum and Difference and Double Angle Identities

Total: 39 marks

You need to show all your steps as you work out your solutions, and make sure your answers are clear (include units, where appropriate). Check the Introduction for the Marking Guidelines that will be used for all assignments and examinations.

1. Without using a calculator, evaluate the following. *(2 × 2 marks each = 4 marks)*

   a) $\cos\frac{27\pi}{10}\cos\frac{19\pi}{20}+\sin\frac{27\pi}{10}\sin\frac{19\pi}{20}$

   b) $\dfrac{\tan\frac{\pi}{18}+\tan\frac{\pi}{9}}{1-\tan\frac{\pi}{18}\tan\frac{\pi}{9}}$

continued

Assignment 6.3: Sum and Difference and Double Angle Identities (continued)

2. Find the exact values of the coordinates of $P\left(\frac{5\pi}{12}\right)$. *(4 marks)*

3. Use a double angle identity to find $\cos\frac{\pi}{9}$, if $\sin\frac{\pi}{18} = 0.17$. *(2 marks)*

continued

Assignment 6.3: Sum and Difference and Double Angle Identities (continued)

4. Prove the following identities.

   a) $\sin(x + y) - \sin(x - y) = 2 \cos x \sin y$ *(2 marks)*

   b) $\tan x = \dfrac{\sin 2x}{1 + \cos 2x}$ *(3 marks)*

continued

Assignment 6.3: Sum and Difference and Double Angle Identities (continued)

c) $\cos 3x = 4\cos^3 x - 3\cos x$ *(3 marks)*

Hint: Write $3x$ as $(2x + x)$ and change everything to functions of x.

continued

Assignment 6.3: Sum and Difference and Double Angle Identities (continued)

5. Solve $\sin 2x + \cos x = 0$ for x, where $0 < x < 2\pi$. *(4 marks)*

6. Given $\sin \alpha = -\frac{4}{5}$ and $\pi < \alpha < \frac{3\pi}{2}$ and $\sin \beta = \frac{12}{13}$ and $\frac{\pi}{2} < \beta < \pi$.

   Find the exact value of

   a) $\sin (\alpha - \beta)$ *(3 marks)*

continued

Assignment 6.3: Sum and Difference and Double Angle Identities (continued)

b) $\cos 2\beta$ *(2 marks)*

c) $\tan 2\beta$ *(4 marks)*

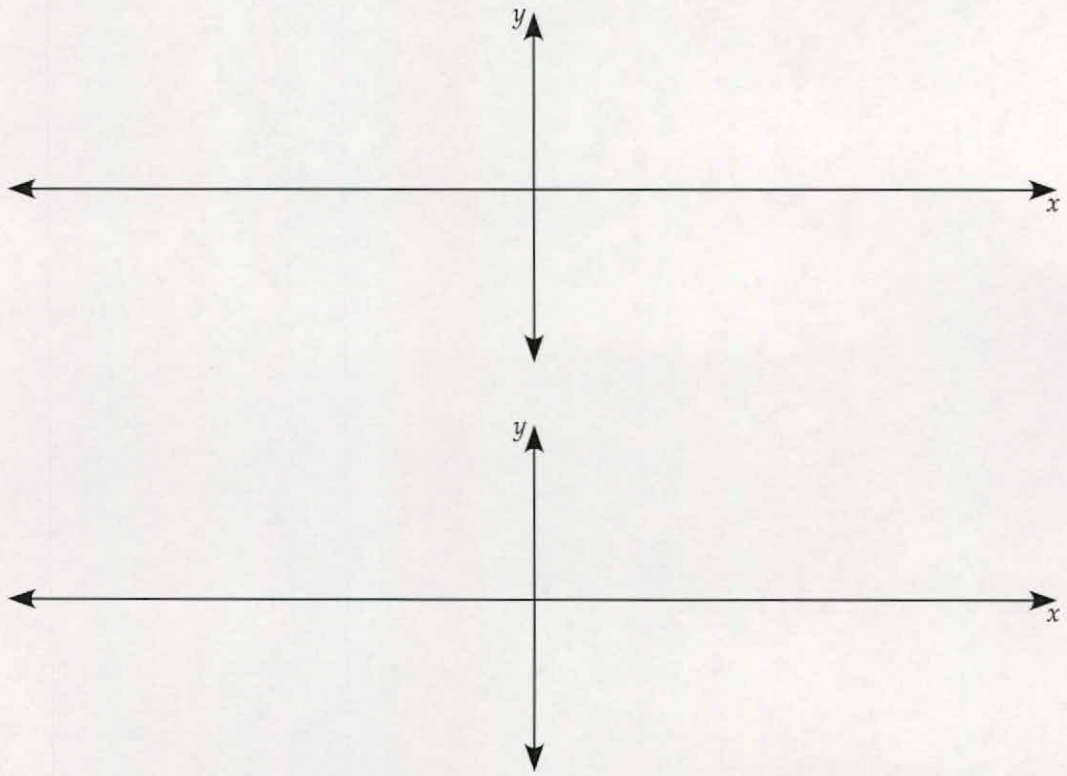
continued

Assignment 6.3: Sum and Difference and Double Angle Identities (continued)

7. Consider the equation $\sin\left(\frac{3\pi}{2} + x\right) = -\cos x$.

   a) Verify the equation holds for $x = \frac{\pi}{3}$. *(1 mark)*

   b) Graph $y = \sin\left(\frac{3\pi}{2} + x\right)$ and $y = -\cos x$ on the coordinate grids below. *(4 marks)*



continued

Assignment 6.3: Sum and Difference and Double Angle Identities (continued)

c) Using the graph you created in (b), do you believe this is a possible identity? *(1 mark)*

d) Verify algebraically that the equation is or is not an identity. *(2 marks)*

Module 6 Summary

In this module, you used the concept of the unit circle, which you learned about in Module 5, to help you solve trigonometric equations and develop trigonometric identities. You had previously learned how to solve linear trigonometric equations, and throughout this module you extended that knowledge to learn how to solve quadratic trigonometric equations.

You then learned about various identities built from the unit circle including Pythagorean identities, sum and difference identities, and double angle identities. Using these identities, you were able to simplify various trigonometric expressions, solve more complex trigonometric equations, and prove other trigonometric identities.

In the next module, you will be learning about exponential and logarithmic functions.

Submitting Your Assignments

It is now time for you to submit Assignments 6.1 to 6.3 to the Distance Learning Unit so that you can receive some feedback on how you are doing in this course. Remember that you must submit all the assignments in this course before you can receive your credit.

Make sure you have completed all parts of your Module 6 assignments and organize your material in the following order:

- ❑ Module 6 Cover Sheet (found at the end of the course Introduction)
- ❑ Assignment 6.1: Solving Trigonometric Equations
- ❑ Assignment 6.2: Using Elementary Identities
- ❑ Assignment 6.3: Sum and Difference and Double Angle identities

For instructions on submitting your assignments, refer to How to Submit Assignments in the course Introduction.

Notes

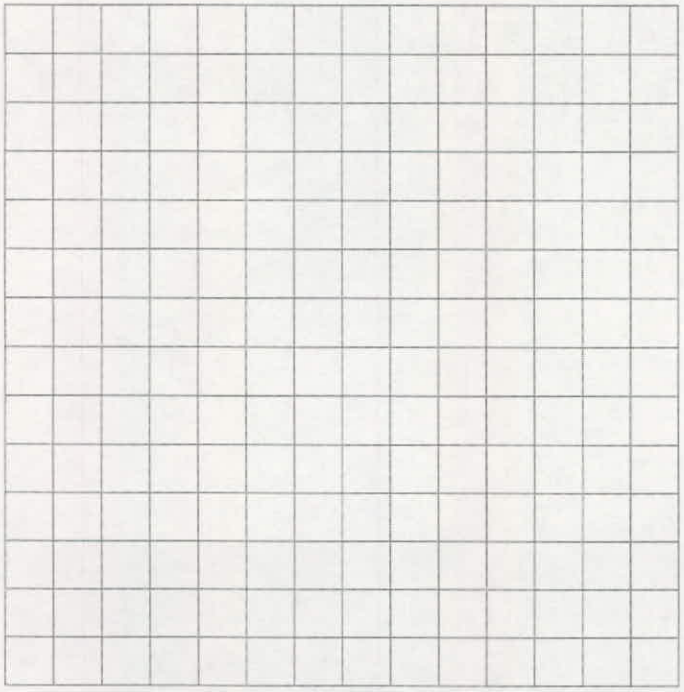
Assignment 7.1

Exponential Functions and Logarithms

Total: 48 marks

You need to show all your steps as you work out your solutions, and make sure your answers are clear (include units, where appropriate). Check the Introduction for the Marking Guidelines that will be used for all assignments and examinations.

1. a) Sketch the graph of $y = 3^x$ and $y = \left(\frac{1}{3}\right)^x$ on the same coordinate system. *(2 marks)*



b) Use transformation language to describe how the graphs are related. *(1 mark)*

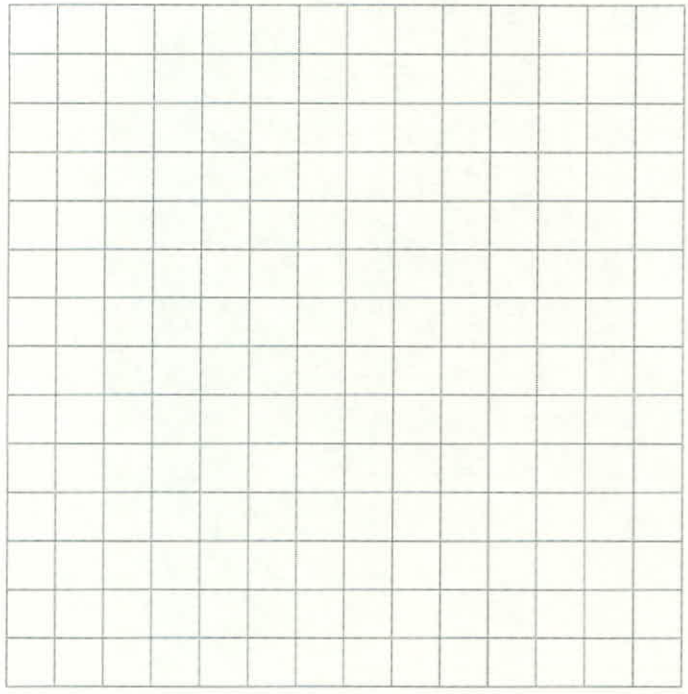
c) Write $y = \left(\frac{1}{3}\right)^x$ in an alternate form. *(1 mark)*

continued

Assignment 7.1: Exponential Functions and Logarithms (continued)

2. Sketch each of the following exponential functions and state the domain, range, y-intercept, and equation of asymptote for each. *(5 × 5 marks each = 25 marks)*

   a) $y = 2^{x-3} + 4$



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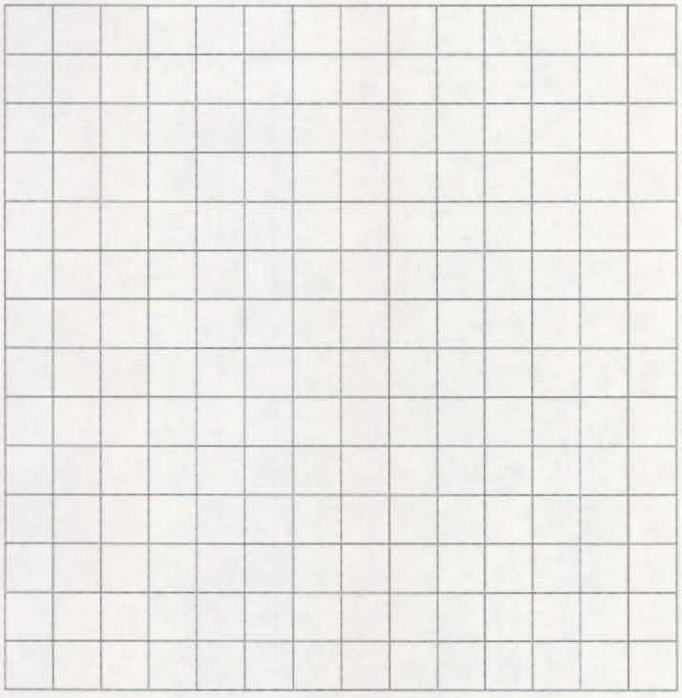
Assignment 7.1: Exponential Functions and Logarithms (continued)

b) $y = \frac{1}{2}\left(3^{x+2}\right)$



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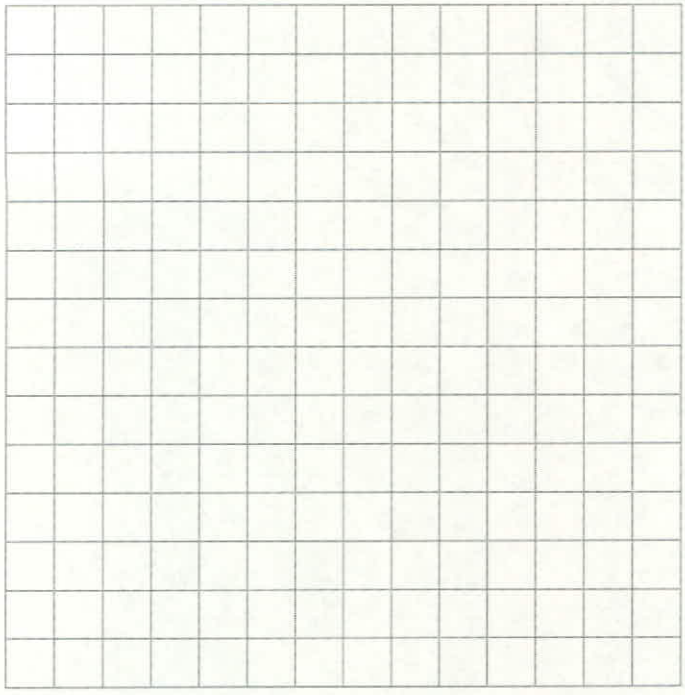
Assignment 7.1: Exponential Functions and Logarithms (continued)

c) $y = -2^{x-1} + 3$



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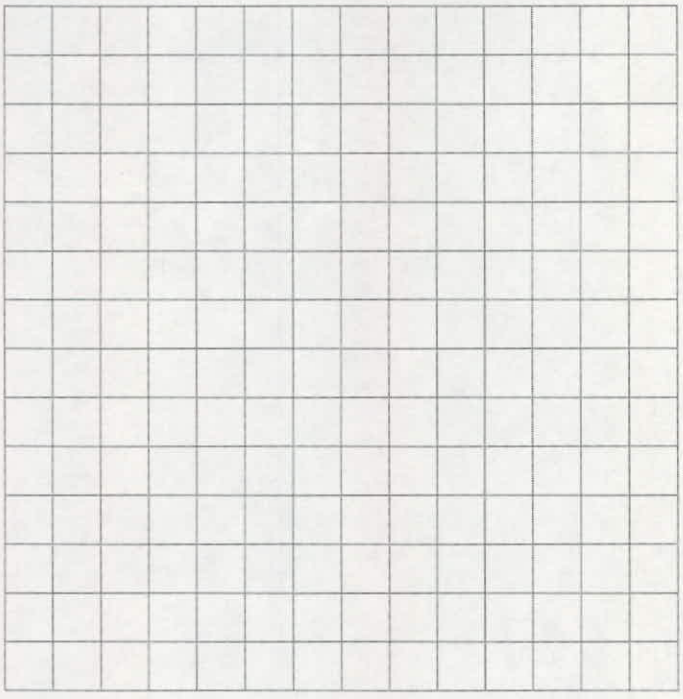
Assignment 7.1: Exponential Functions and Logarithms (continued)

d) $y = 2(3^{-x-1})$



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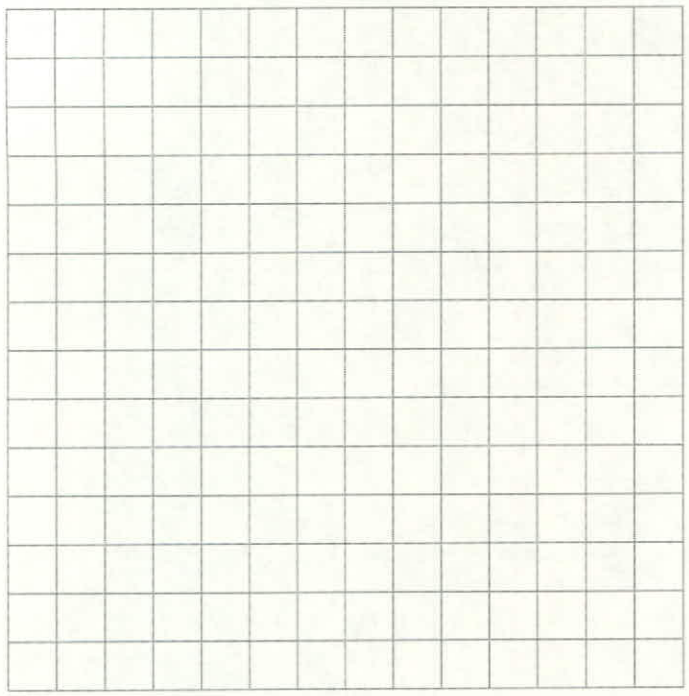
Assignment 7.1: Exponential Functions and Logarithms (continued)

e) $y = -e^{x+2}$



continued

Assignment 7.1: Exponential Functions and Logarithms (continued)

3. Express each of the following in exponential form. *(4 × 0.5 mark each = 2 marks)*

   a) $\log_7 49 = 2$

   b) $\log \frac{1}{1000} = -3$

   c) $\log_{\frac{1}{4}} \frac{1}{64} = 3$

   d) $\ln 2 = x$

continued

Assignment 7.1: Exponential Functions and Logarithms (continued)

4. Express each of the following in logarithmic form. *(4 × 0..5 mark each = 2 marks)*

   a) $6^3 = 216$

   b) $\left(\frac{1}{3}\right)^{-5} = 243$

   c) $512 = 8^3$

   d) $e^5 = x$

continued

5. Solve for the variable.

a) $\log_4 x = 2$ *(1 mark)*

b) $\log_x 36 = -2$ *(2 marks)*

c) $\log_9 \sqrt{3} = x$ *(2 marks)*

continued

d) $\log_8 x = -\frac{2}{3}$ *(2 marks)*

e) $\log x = 4$ *(1 mark)*

f) $\log_3 \frac{1}{81} = x$ *(2 marks)*

g) $\ln x = 3$ *(1 mark)*

continued

Assignment 7.1: Exponential Functions and Logarithms (continued)

6. Estimate the following logarithms to one decimal place without using a calculator. Show your work. *(2 × 2 marks each = 4 marks)*

   a) $\log_4 71$

   b) $\log_3 35$

Notes

Assignment 7.2

Dealing with Logarithms

Total: 46 marks

You need to show all your steps as you work out your solutions, and make sure your answers are clear (include units, where appropriate). Check the Introduction for the Marking Guidelines that will be used for all assignments and examinations.

1. Write as a single logarithm. *(2 × 3 marks each = 6 marks)*

   a) $\log_3 A - 2\log_3 B + \frac{1}{2}\log_3 C$

   b) $5\log m - 4\log n - \frac{1}{2}\log p$

2. Express the following expression as a single logarithm. Simplify, if possible. *(2 marks)*

   $\log_6 4 + \log_6 9 + \log_6 6$

continued

Assignment 7.2: Dealing with Logarithms (continued)

3. Expand as a sum and/or difference of individual logarithmic expressions, and simplify if possible.

a) $\log_2\left(\frac{\sqrt{x}(x-5)}{y^3}\right)$ *(3 marks)*

b) $\log\left(\frac{\sqrt{A}}{B^2\sqrt[3]{C}}\right)$ *(3 marks)*

c) $\log_3\left((27^3)\sqrt[3]{\frac{x^2}{y}}\right)$ *(5 marks)*

continued

Assignment 7.2: Dealing with Logarithms (continued)

4. Given:

   $\log_b 3 = A$

   $\log_b 4 = B$

   $\log_b 7 = C$

   Use the three equations given above to write an expression for the following.

   a) $\log_b 12$ *(2 marks)*

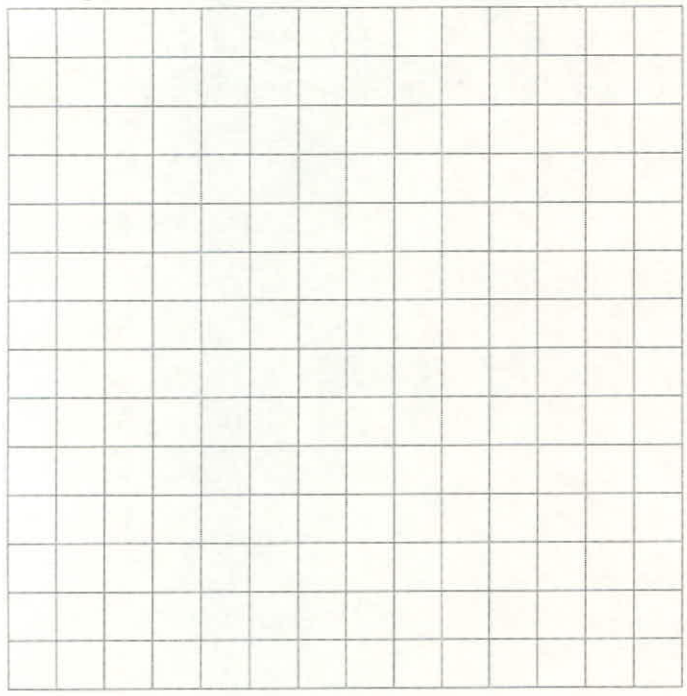
   b) $\log_b \frac{21}{4}$ *(3 marks)*

continued

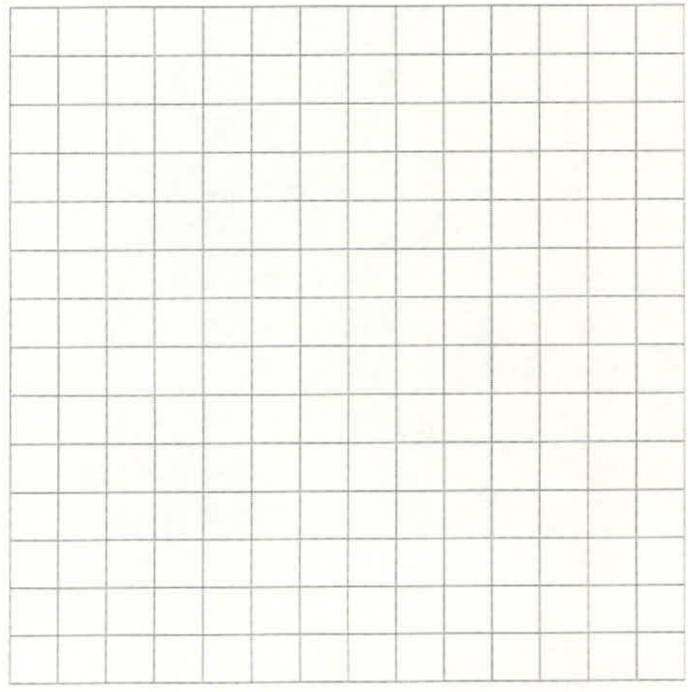
Assignment 7.2: Dealing with Logarithms (continued)

5. Sketch each of the following logarithmic functions and state the domain, range, and equation of asymptote for each. *(3 × 4 marks each = 12 marks)*

   a) $y = \log_5 x - 1$



   b) $y = -\log (x + 4)$



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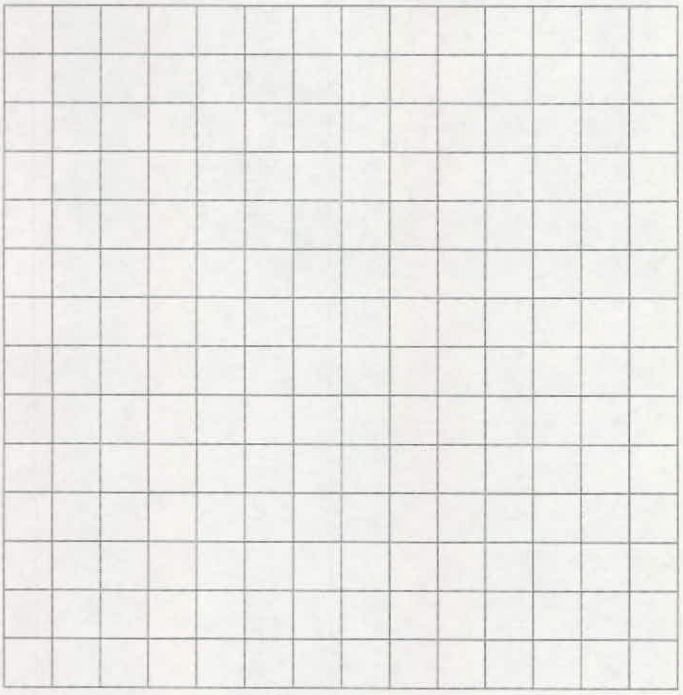
Assignment 7.2: Dealing with Logarithms (continued)

c) $y = \log_2 (x - 3) + 4$



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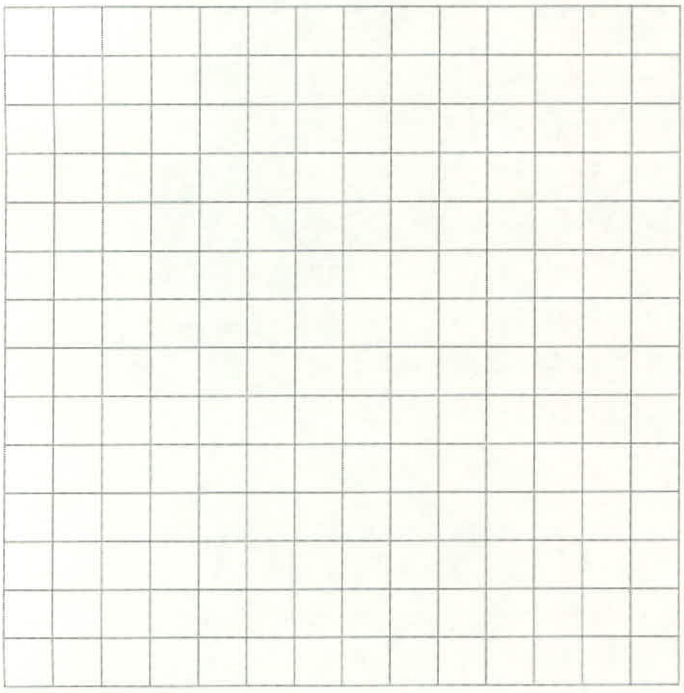
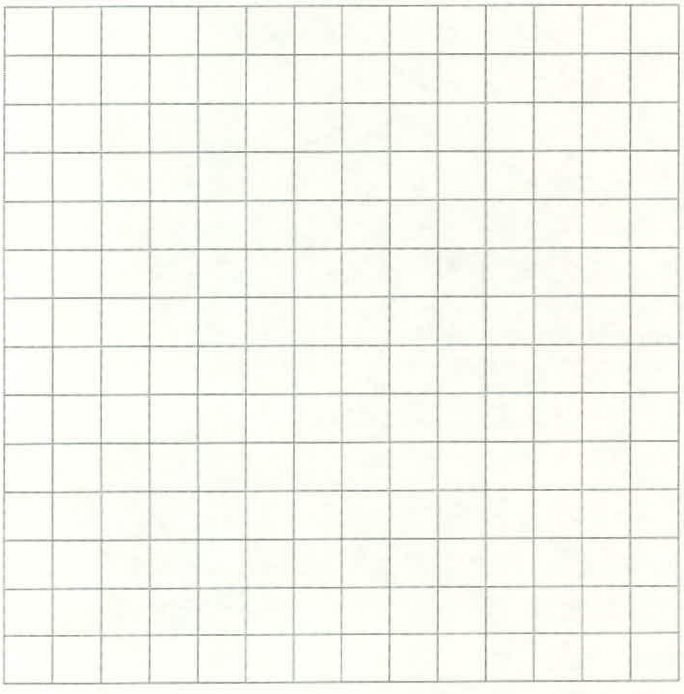
Assignment 7.2: Dealing with Logarithms (continued)

6. Sketch $f(x) = 7^x$ and $f^{-1}(x)$ on the coordinate grids given below. State the domain, range, and equations of asymptotes of both $f(x)$ and $f^{-1}(x)$. *(5 marks)*





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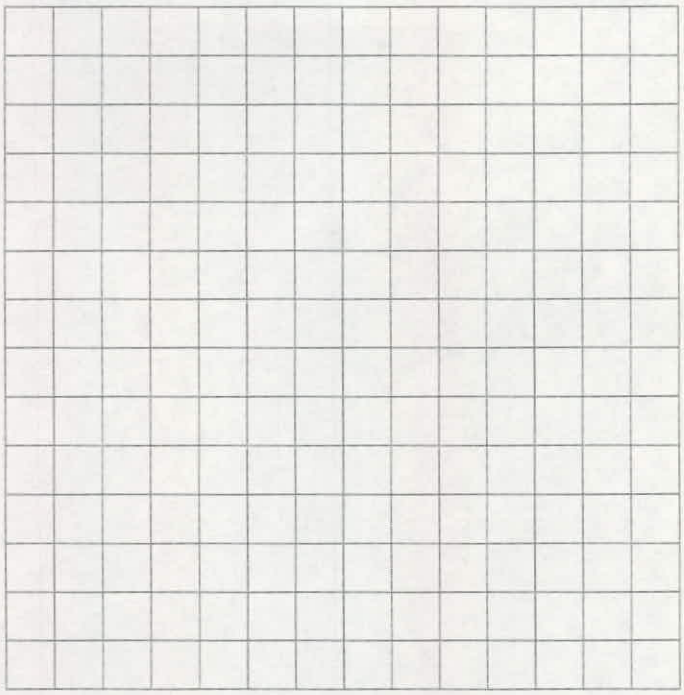
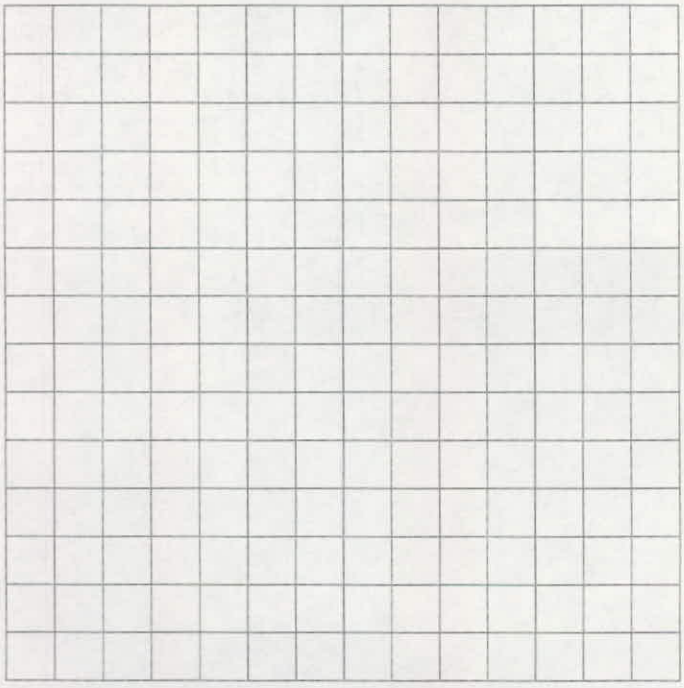
Assignment 7.2: Dealing with Logarithms (continued)

7. Sketch $f(x) = \log_4 (x + 3)$ and $g(x) = 4^x - 3$ on the coordinate grids given below. Give two ways in which these functions are related. *(5 marks)*





Notes

Assignment 7.3

Solving Exponential and Logarithmic Equations

Total: 37 marks

You need to show all your steps as you work out your solutions, and make sure your answers are clear (include units, where appropriate). Check the Introduction for the Marking Guidelines that will be used for all assignments and examinations.

1. Solve the following exponential equations without using a calculator.
   (3 × 1 mark each = 3 marks)

   a) $2^{\frac{x}{3}} = 32$

   b) $2^{3x+5} = \frac{1}{16}$

   c) $4^{2x-6} = 1$

continued

Assignment 7.3: Solving Exponential and Logarithmic Equations (continued)

2. Solve the following exponential equations. State your answer to two decimal places. *(3 × 4 marks each = 12 marks)*

   a) $2\left(5^{x}\right) = 751$

   b) $3e^{2x-5} = 25$

continued

Assignment 7.3: Solving Exponential and Logarithmic Equations (continued)

c) $4\left(3^{x}\right) = 91$

3. Find these logarithms by using a calculator. State your answer to four decimal places. *(3 × 1 mark each = 3 marks)*

   a) log 6

   b) ln 3

   c) log (−0.123)

continued

Assignment 7.3: Solving Exponential and Logarithmic Equations (continued)

4. Solve the following logarithmic equations. Remember to check for extraneous roots! *(3 × 3 marks each = 9 marks)*

   a) $\log_3 (x) + \log_3 (x - 2) = 1$

   b) $\log_2 (x - 4) - \log_2 (x - 3) = 1$

continued

Assignment 7.3: Solving Exponential and Logarithmic Equations (continued)

c) $\log_6 \left(x^2 - 16\right) - \log_6 x = \log_6 6$

continued

Assignment 7.3: Solving Exponential and Logarithmic Equations (continued)

5. A radioactive substance is decaying according to the formula $y = Ae^{-0.2t}$, where y is the amount of material remaining after t years.

   a) If the initial amount is $A = 80$ grams, how much remains after three years? *(1 mark)*

   b) Find the half-life of this substance. *(3 marks)*

continued

Assignment 7.3: Solving Exponential and Logarithmic Equations (continued)

6. Julie invests $100 at 10%.
   a) How much is the investment worth after one year? *(1 mark)*

   b) How long will it take for the money to double itself? *(3 marks)*

continued

Assignment 7.3: Solving Exponential and Logarithmic Equations (continued)

7. Use your knowledge of logarithms to answer the following questions.
   (2 × 1 mark each = 2 marks)

   a) How many times more energy is contained within an earthquake that is rated a 7 on the Richter scale than an earthquake that is rated a 1 on the Richter scale?

   b) If a certain brand of dish soap has a pH level of 8, how many times more acidic is lime juice that has a pH level of 3.5?

Module 7 Summary

In this module, you learned about exponential and logarithmic functions. First, you built on your knowledge of exponents to graph exponential functions. You then learned about the inverse functions of exponential functions called logarithmic functions. You were able to graph these functions using two methods—the method of first graphing the inverse exponential function and then reflecting that function through the line $y = x$, and the method of using transformations of the standard logarithmic function.

Just as there are rules for simplifying exponents, there are also rules for simplifying logarithmic expressions. You learned about the logarithmic theorems that helped you to solve both logarithmic and exponential equations. Once you were able to solve logarithmic and exponential equations, you could solve many application problems. As you saw throughout Lesson 7, exponential and logarithmic functions have many applications in nature and financial situations.

In the next module, you are going to be studying rational and radical functions.

Submitting Your Assignments

It is now time for you to submit Assignments 7.1 to 7.3 to the Distance Learning Unit so that you can receive some feedback on how you are doing in this course. Remember that you must submit all the assignments in this course before you can receive your credit.

Make sure you have completed all parts of your Module 7 assignments and organize your material in the following order:

- ❑ Module 7 Cover Sheet (found at the end of the course Introduction)
- ❑ Assignment 7.1: Exponential Functions and Logarithms
- ❑ Assignment 7.2: Dealing with Logarithms
- ❑ Assignment 7.3: Solving Exponential and Logarithmic Equations

For instructions on submitting your assignments, refer to How to Submit Assignments in the course Introduction.

Notes

Assignment 8.1

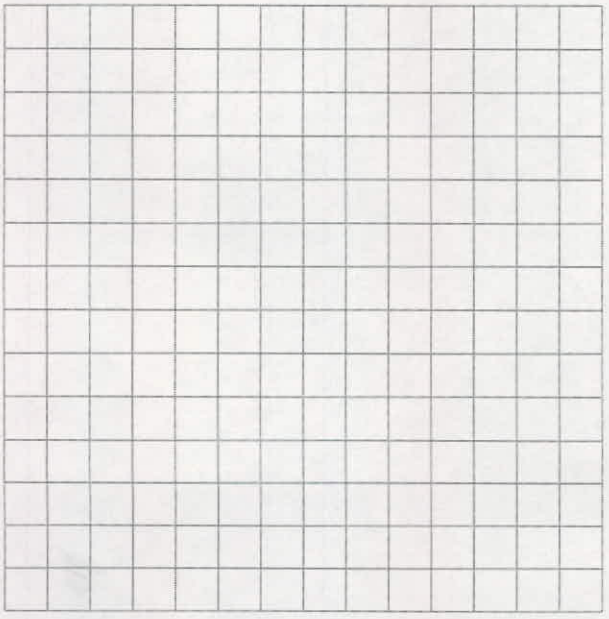
Radical Functions

Total: 34 marks

You need to show all your steps as you work out your solutions, and make sure your answers are clear (include units, where appropriate). Check the Introduction for the Marking Guidelines that will be used for all assignments and examinations.

1. Graph the following functions using transformations. State the domain and range of each function. *(4 × 3 marks each = 12 marks)*

   a) $y = 2\sqrt{x+2} - 5$



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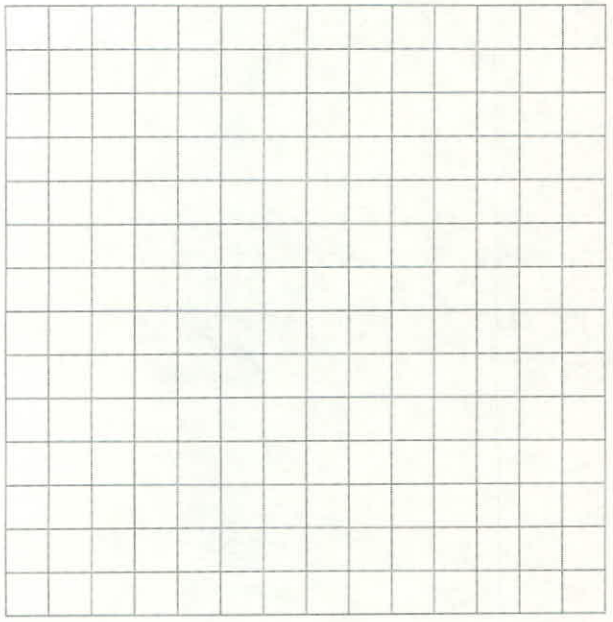
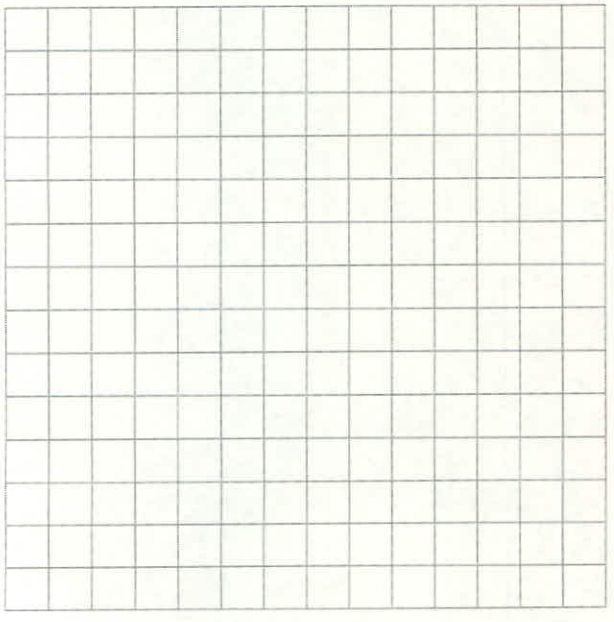
Assignment 8.1: Radical Functions (continued)

b) $y = \sqrt{-\frac{1}{2}x + 3}$



c) $y = -2\sqrt{-(x+1)}$



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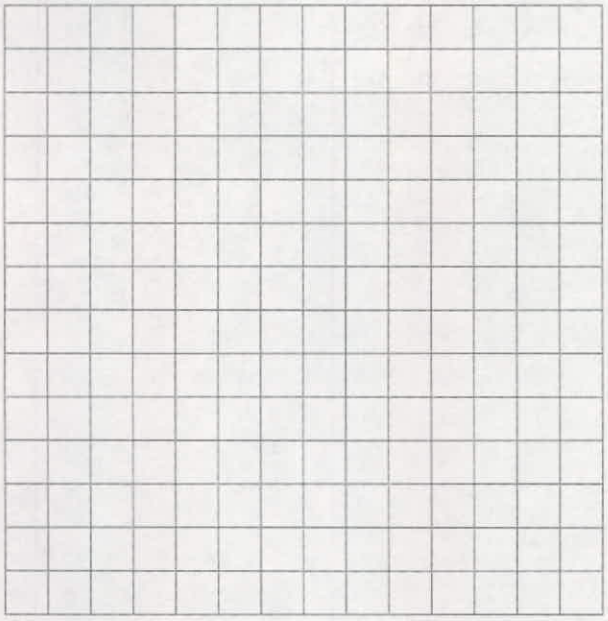
Assignment 8.1: Radical Functions (continued)

d) $y = -\sqrt{\left(\frac{1}{3}\right)(x-3)} - 5$



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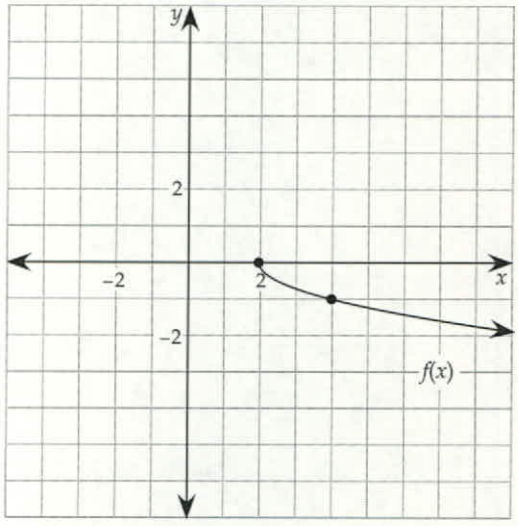
Assignment 8.1: Radical Functions (continued)

2. Determine the equations of the functions that correspond to the following graphs. *(2 × 2 marks each = 4 marks)*

a)



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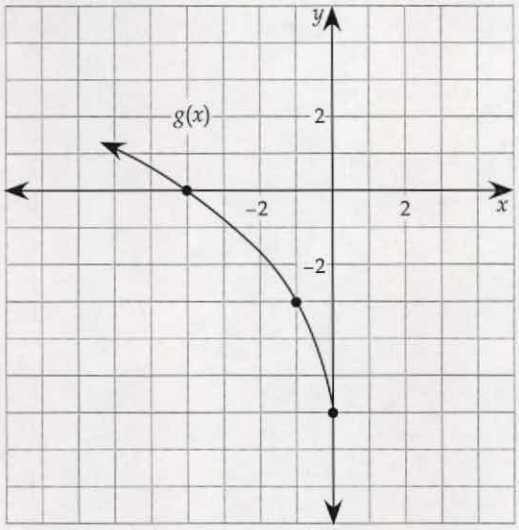
Assignment 8.1: Radical Functions (continued)

b)



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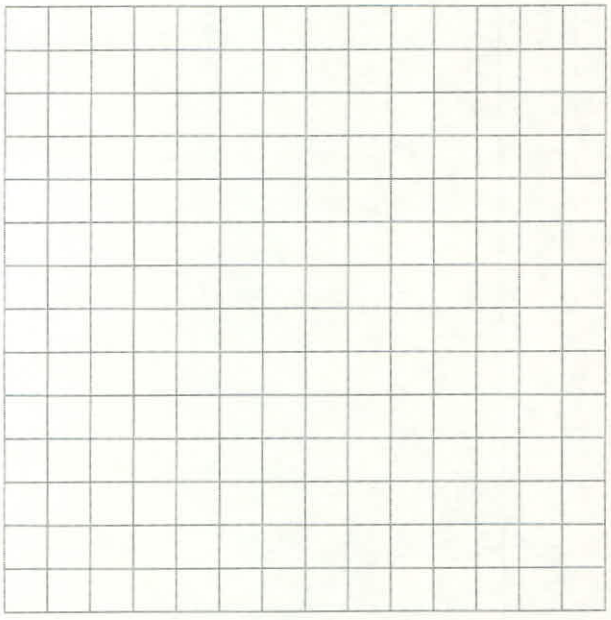
Assignment 8.1: Radical Functions (continued)

3. Express the following function, $f(x)$, algebraically as a mapping from $y = \sqrt{x}$. Sketch the function $f(x)$, and state the domain and range. *(5 marks)*

$$f(x) = -\frac{1}{2}\sqrt{-(x+2)} - 3$$

Graphically:



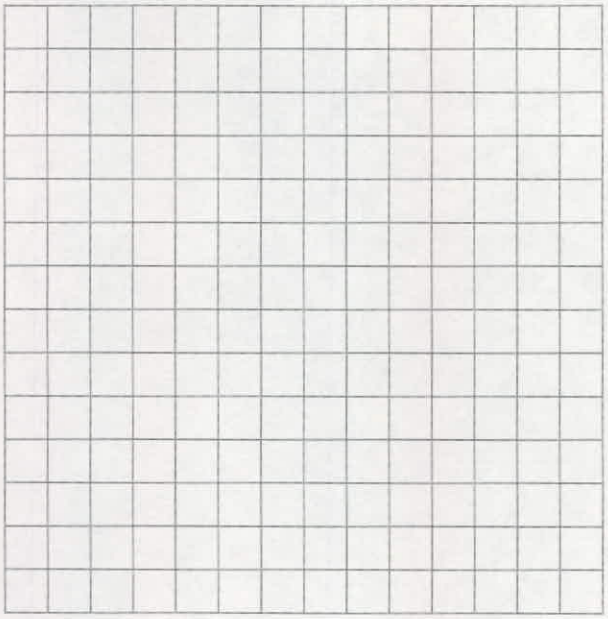
Algebraically:

continued

Assignment 8.1: Radical Functions (continued)

4. Graph the square root of the following function. State the domain and range of the square root of the function. *(3 marks)*

   $y = -2x + 4$



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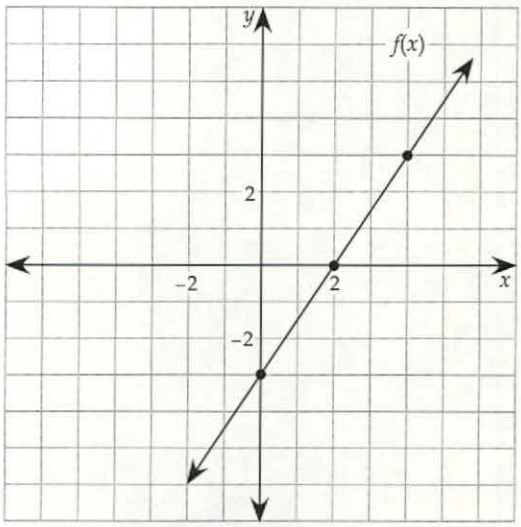
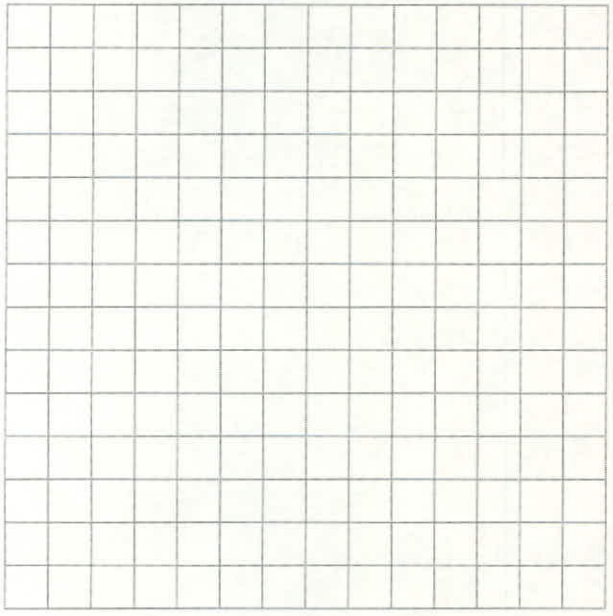
Assignment 8.1: Radical Functions (continued)

5. Graph the square root of each of the functions below. *(3 × 2 marks each = 6 marks)*

a)





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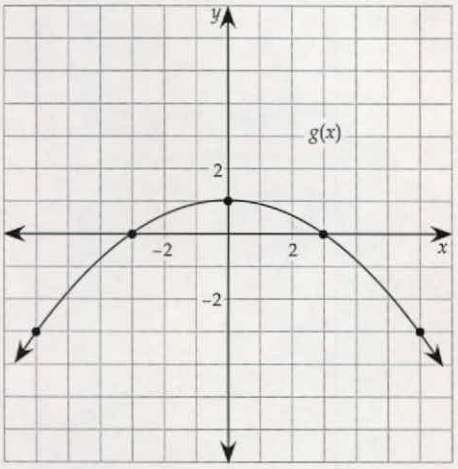
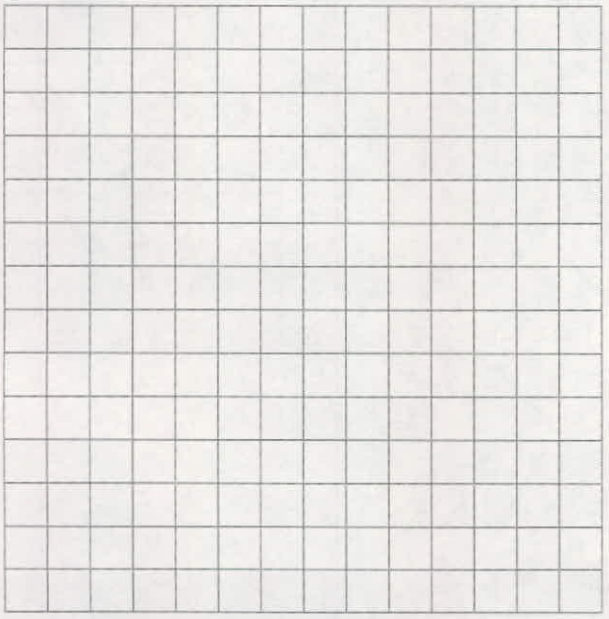
Assignment 8.1: Radical Functions (continued)

b)





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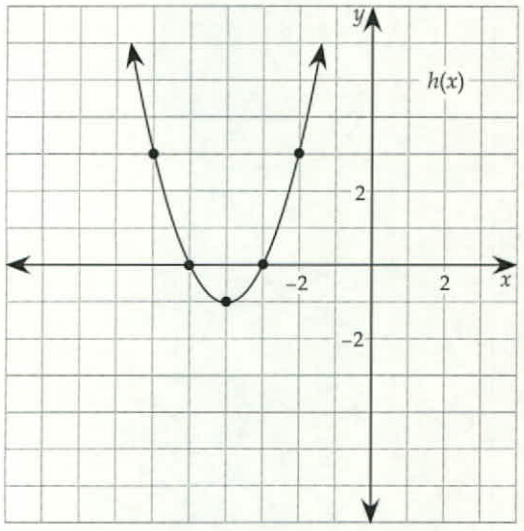
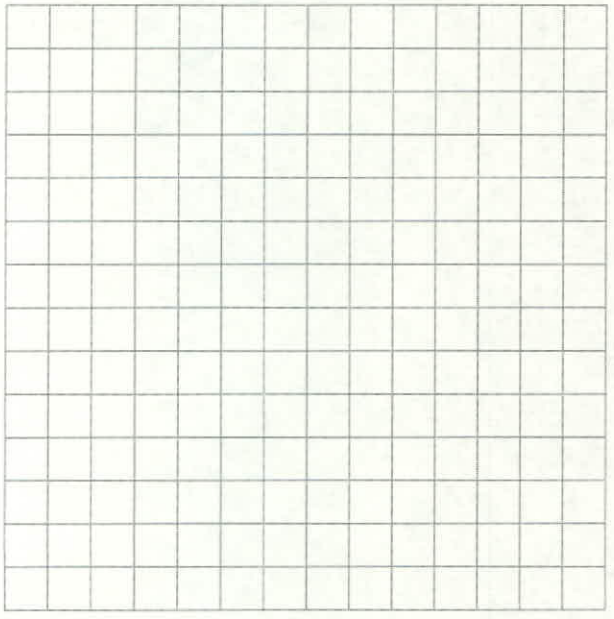
Assignment 8.1: Radical Functions (continued)

c)





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Assignment 8.1: Radical Functions (continued)

6. Compare the domain and range of $f(x) = 2x - 6$ with the domain of $y = \sqrt{f(x)}$. Explain why the domains and ranges of $y = f(x)$ and $y = \sqrt{f(x)}$ are different. *(4 marks)*

Notes

Assignment 8.2

Solving Radical Equations

Total: 13 marks

You need to show all your steps as you work out your solutions, and make sure your answers are clear (include units, where appropriate). Check the Introduction for the Marking Guidelines that will be used for all assignments and examinations.

1. Consider the equation $2\sqrt{x+3} = 6$.

   a) Determine the roots of the above equation algebraically. *(2 marks)*

continued

Assignment 8.2: Solving Radical Equations (continued)

b) Determine the x-intercept of the corresponding function $y = 2\sqrt{x+3} - 6$. *(2 marks)*

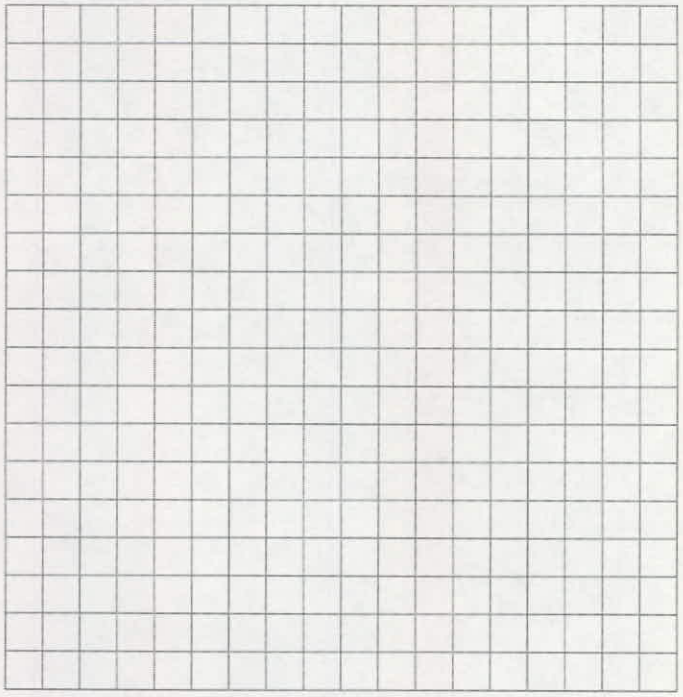
c) Explain the relationship between the roots of $2\sqrt{x+3} = 6$ and the x-intercepts of $y = 2\sqrt{x+3} - 6$. *(1 mark)*

continued

Assignment 8.2: Solving Radical Equations (continued)

2. Estimate the solution to the following radical equation graphically.

$\sqrt{3x-6} = 2x - 7$ *(4 marks)*



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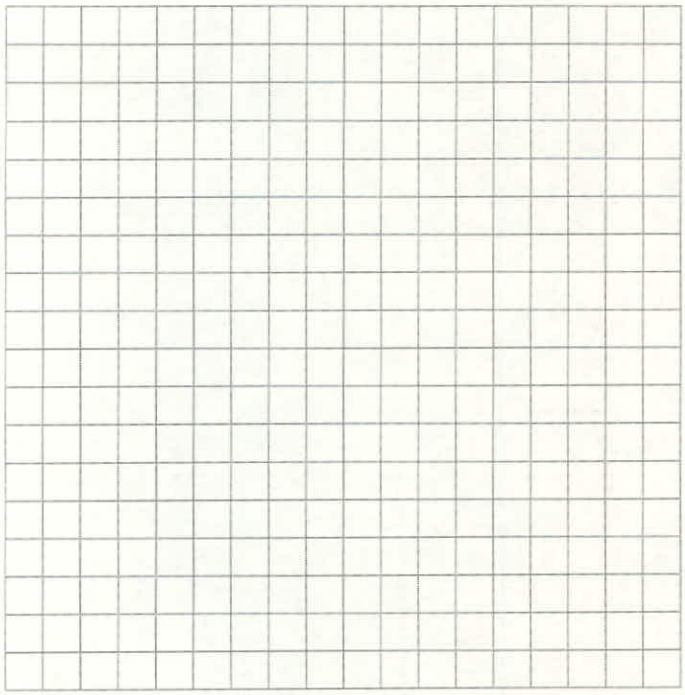
Assignment 8.2: Solving Radical Equations (continued)

3. Estimate the solution to the following radical equation graphically.

$\sqrt{4x-8} = \frac{1}{5}x$ *(4 marks)*



Assignment 8.3

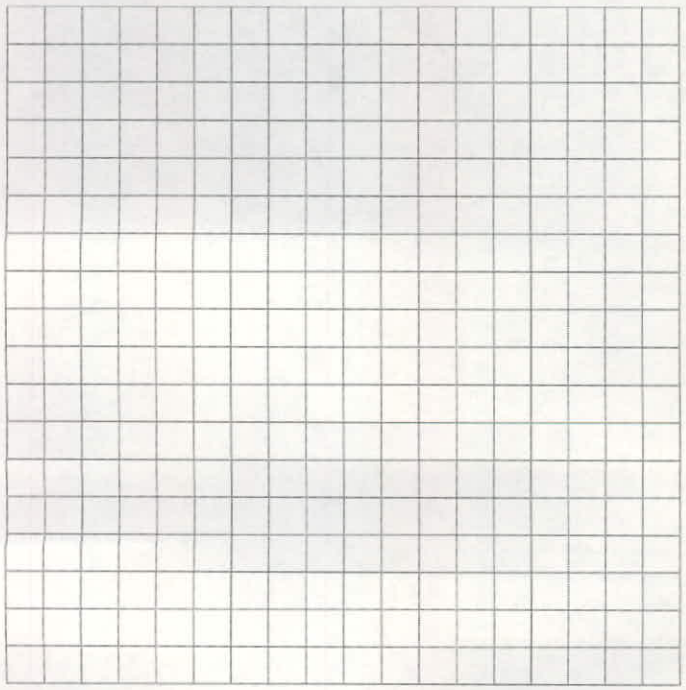
Rational Functions

Total: 34 marks

You need to show all your steps as you work out your solutions, and make sure your answers are clear (include units, where appropriate). Check the Introduction for the Marking Guidelines that will be used for all assignments and examinations.

1. Graph the following functions. Pay attention to whether each graph should have a point of discontinuity or a vertical asymptote (or both).

   a) $f(x) = \dfrac{2}{x+6} - 5$ *(3 marks)*



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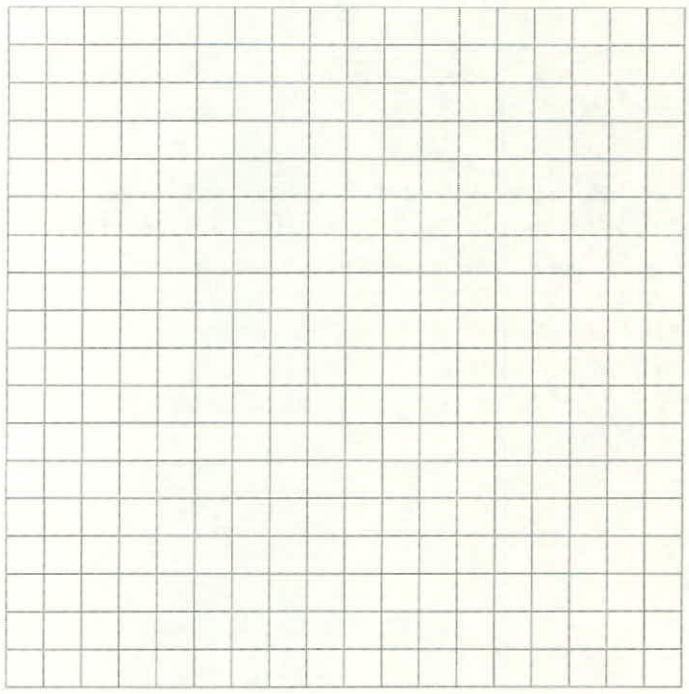
Assignment 8.3: Rational Functions (continued)

b) $f(x) = \dfrac{x}{x^2 - 2x - 8}$ *(5 marks)*



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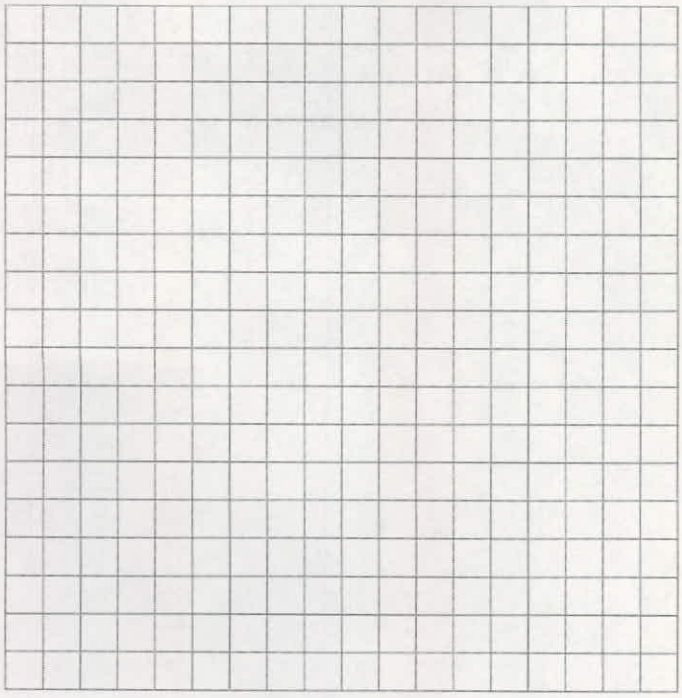
Assignment 8.3: Rational Functions (continued)

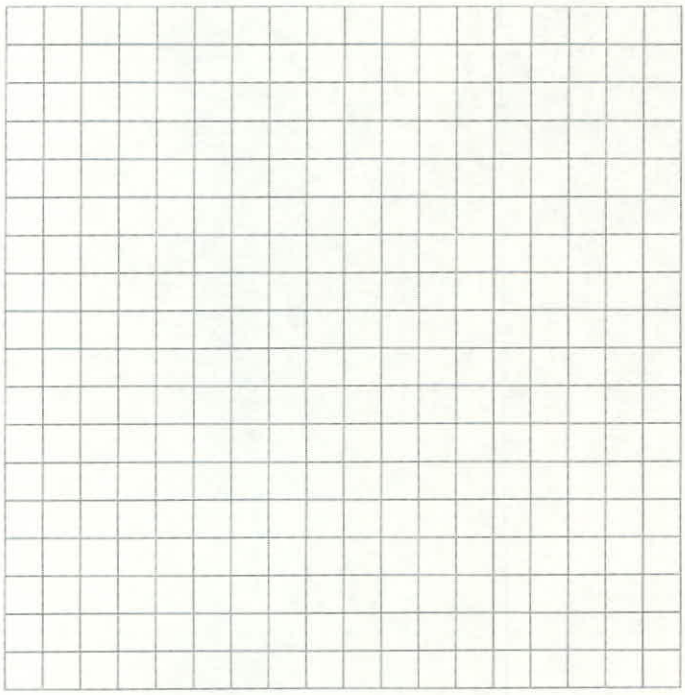
c) $f(x) = \dfrac{x+5}{x-5}$ *(3 marks)*



continued

Assignment 8.3: Rational Functions (continued)

d) $f(x) = \dfrac{x^2 - 11x + 28}{x - 4}$ *(2 marks)*



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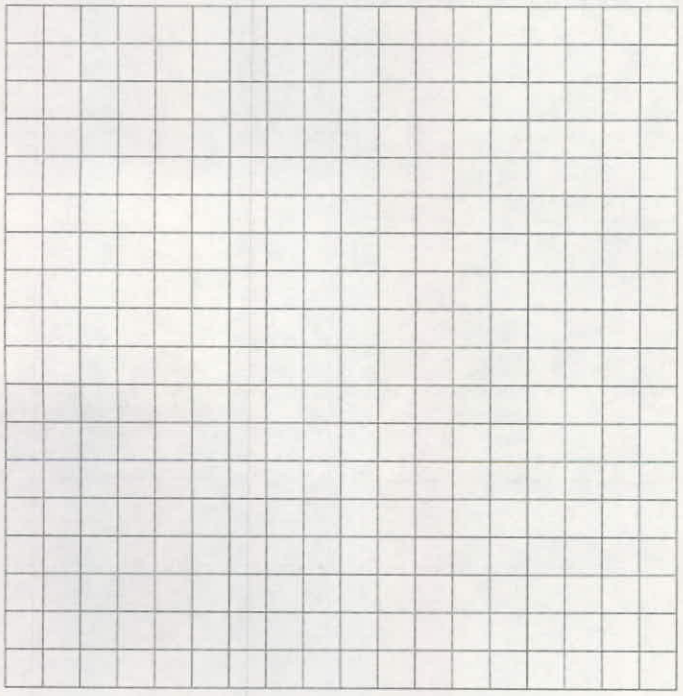
Assignment 8.3: Rational Functions (continued)

e) $f(x) = \dfrac{x^2}{x^2 - 2x - 15}$ *(5 marks)*



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Assignment 8.3: Rational Functions (continued)

2. Compare the functions $y = \frac{1}{x^2 - 2x - 35}$ and $y = \frac{x + 6}{2x^2 + 13x + 6}$ using the following chart. What do the functions have in common? Explain. *(6 marks)*

	$y = \frac{1}{x^2 - 2x - 35} =$	$y = \frac{x + 6}{2x^2 + 13x + 6} =$
x-intercept		
y-intercept		
vertical asymptote		
horizontal asymptote		
points of discontinuity		

continued

Assignment 8.3: Rational Functions (continued)

3. Match each function to its corresponding graph, and explain your reasoning. *(6 × 1 mark each = 6 marks)*

a) $y = \dfrac{5}{x-3}$

b) $y = \dfrac{x-3}{x-3}$

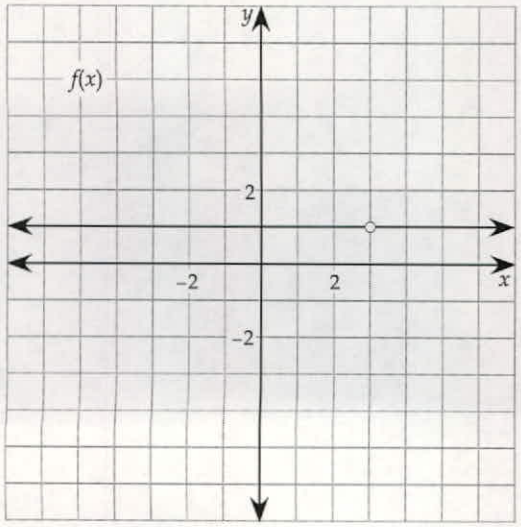
c) $y = \dfrac{x-6}{x^2-9x+18}$

d) $y = \dfrac{1}{x^2-9x+18}$

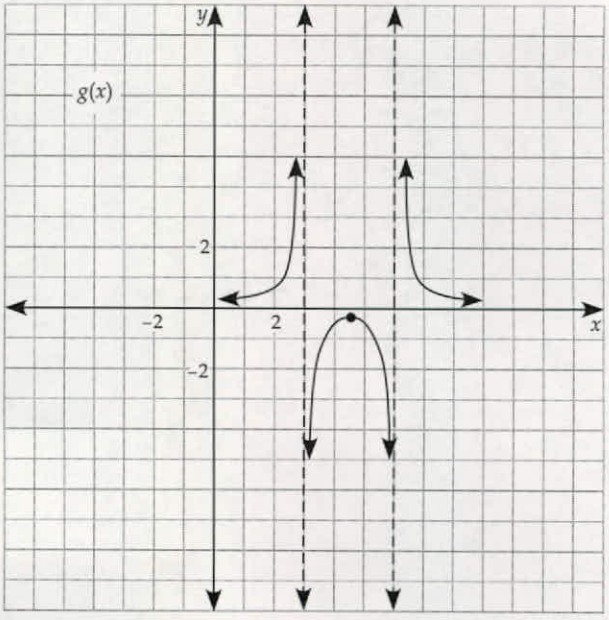
e) $y = \dfrac{x^2-2x-24}{x^2-9x+18}$

f) $y = \dfrac{x-6}{x-3}$

i)



ii)



continued

Assignment 8.3: Rational Functions (continued)

a) $y = \dfrac{5}{x-3}$

b) $y = \dfrac{x-3}{x-3}$

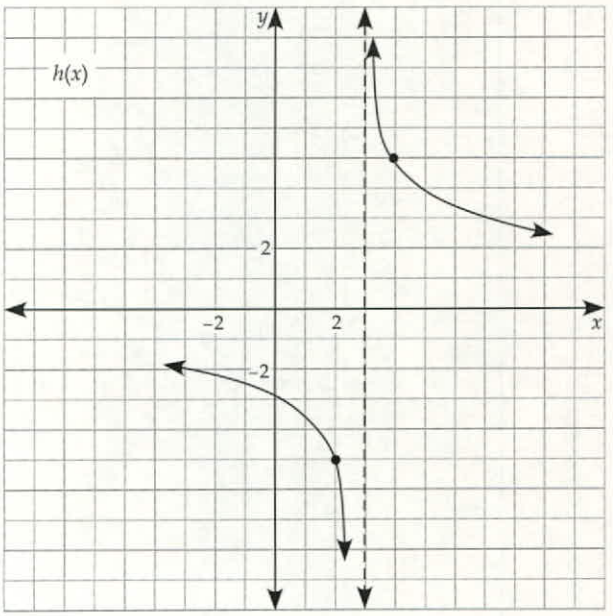
c) $y = \dfrac{x-6}{x^2-9x+18}$

d) $y = \dfrac{1}{x^2-9x+18}$

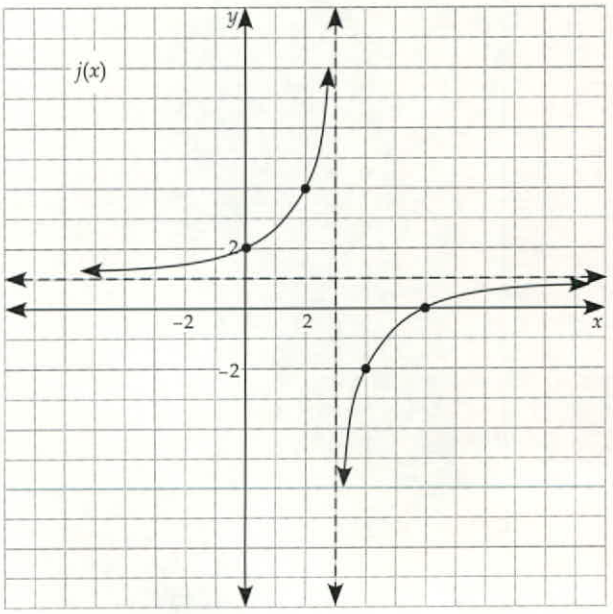
e) $y = \dfrac{x^2-2x-24}{x^2-9x+18}$

f) $y = \dfrac{x-6}{x-3}$

iii)



iv)



continued

Assignment 8.3: Rational Functions (continued)

a) $y = \dfrac{5}{x-3}$

b) $y = \dfrac{x-3}{x-3}$

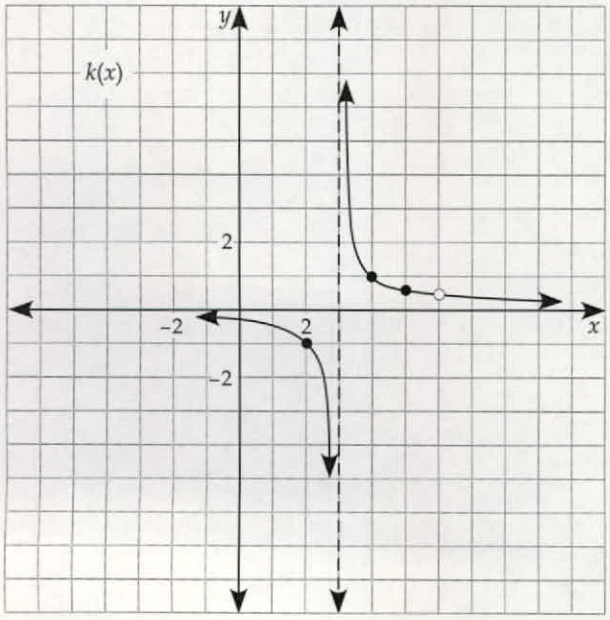
c) $y = \dfrac{x-6}{x^2-9x+18}$

d) $y = \dfrac{1}{x^2-9x+18}$

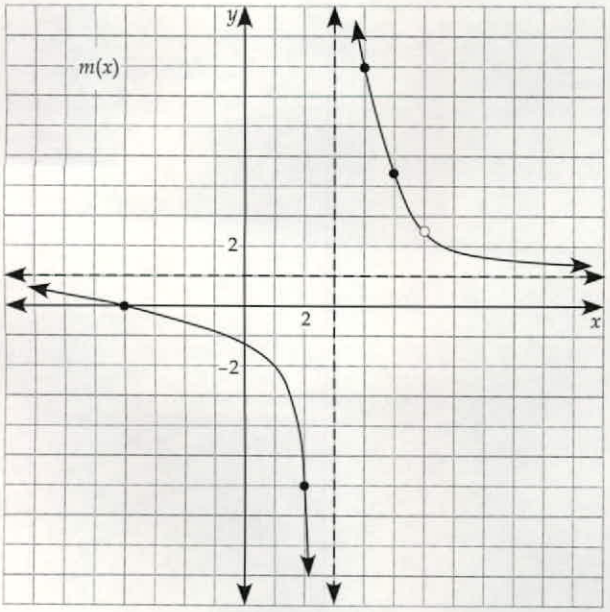
e) $y = \dfrac{x^2-2x-24}{x^2-9x+18}$

f) $y = \dfrac{x-6}{x-3}$

v)



vi)

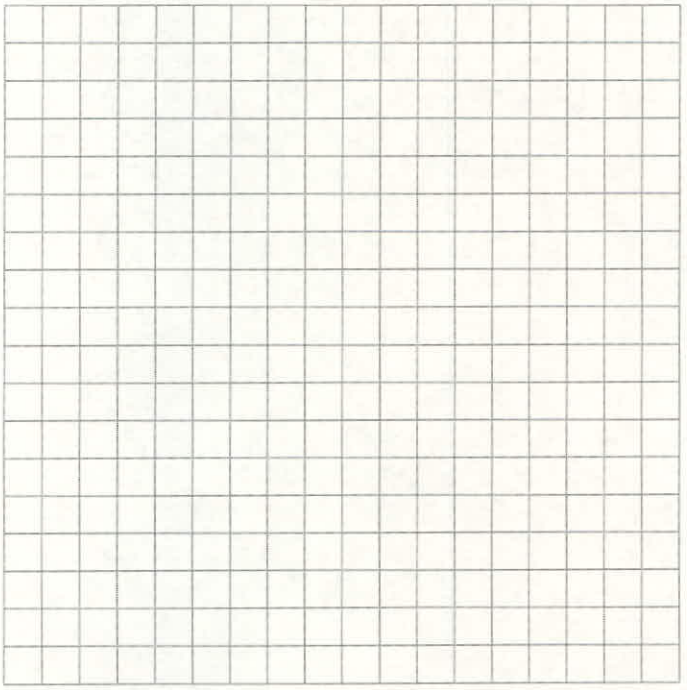


continued

Assignment 8.3: Rational Functions (continued)

4. Find the approximate solution to the following equation graphically.

$$\frac{1}{x+6} = -1 \qquad \textit{(4 marks)}$$



Module 8 Summary

Congratulations! You have completed this course!

In this module, you learned about radical and rational functions and how to solve radical and rational equations. Radical functions are functions that contain a radical sign, similar to radical equations. Rational functions are functions that contain polynomials in their numerator and in their denominator, similar to rational equations.

You learned how to graph both radical and rational functions by using transformations. You also learned how to graph rational functions by analyzing the functions to determine their properties, such as asymptotes, intercepts, and points of discontinuity.

To solve radical and rational equations, you built on your knowledge from previous courses in order to solve these equations graphically.

Submitting Your Assignments

It is now time for you to submit Assignments 8.1 to 8.3 to the Distance Learning Unit so that you can receive some feedback on how you are doing in this course. Remember that you must submit all the assignments in this course before you can receive your credit.

Make sure you have completed all parts of your Module 8 assignments and organize your material in the following order:

- ❑ Module 8 Cover Sheet (found at the end of the course Introduction)
- ❑ Assignment 8.1: Radical Functions
- ❑ Assignment 8.2: Solving Radical Equations
- ❑ Assignment 8.3: Rational Functions

For instructions on submitting your assignments, refer to How to Submit Assignments in the course Introduction.

Final Examination

Congratulations, you have finished Module 8 in the course. The final examination is out of 100 marks and worth 25% of your final mark. In order to do well on this examination, you should review all of your learning activities and assignments from Modules 1 to 8.

You will complete this examination while being supervised by a proctor. You should already have made arrangements to have the examination sent to the proctor from the Distance Learning Unit. If you have not yet made arrangements to write it, then do so now. The instructions for doing so are provided in the Introduction to this module.

You will need to bring the following items to the examination: pens/pencils (2 or 3 of each), blank paper, a ruler, a scientific calculator, and your Examination Resource Sheet. A maximum of 3 hours is available to complete your final examination. When you have completed it, the proctor will then forward it for assessment. Good luck!

At this point you will also have to combine your resource sheets from Modules 1 to 8 onto one $8\frac{1}{2}'' \times 11''$ paper (you may use both sides). Be sure you have all the formulas, definitions, and strategies that you think you will need. This paper can be brought into the examination with you.

Examination Review

You are now ready to begin preparing for your final examination. Please review the content, learning activities, and assignments from Modules 1 to 8.

The final practice examination is also an excellent study aid for reviewing Modules 1 to 8.

You will learn what types of questions will appear on the examination and what material will be assessed. Remember, your mark on the final examination determines 25% of your final mark in this course and you will have 3 hours to complete the examination.

Final Practice Examination and Answer Key

To help you succeed in your examination, a practice examination can be found in the learning management system (LMS). The final practice examination is very similar to the actual examination that you will be writing. The answer key is also included so that, when you have finished writing the practice examination, you can check your answers. This will give you the confidence that you need to do well on your examination. If you do not have access to the Internet, contact the Distance Learning Unit at 1-800-465-9915 to get a copy of the practice examination and the answer key.

To get the most out of your final practice examination, follow these steps:

1. Study for the final practice examination as if it were an actual examination.
2. Review those learning activities and assignments from Modules 1 to 8 that you found the most challenging. Reread those lessons carefully and learn the concepts.
3. Contact your learning partner and your tutor/marker if you need help.
4. Review your lessons from Modules 1 to 8, including all of your notes, learning activities, and assignments.
5. Use your module resource sheets to make a draft of your Final Examination Resource Sheet. You can use both sides of an 8½" by 11" piece of paper.
6. Bring the following to the final practice examination: pens/pencils (2 or 3 of each), blank paper, a ruler, a scientific calculator, and your Final Examination Resource Sheet.
7. Write your final practice examination as if it were an actual examination. In other words, write the entire examination in one sitting, and don't check your answers until you have completed the entire examination. Remember that the time allowed for writing the final examination is 3 hours.
8. Once you have completed the entire practice examination, check your answers against the answer key. Review the questions that you got wrong. For each of those questions, you will need to go back into the course and learn the things that you have missed.
9. Go over your resource sheet. Was anything missing or is there anything that you didn't need to have on it? Make adjustments to your Final Examination Resource Sheet. Once you are happy with it, make a photocopy that you can keep.

Notes